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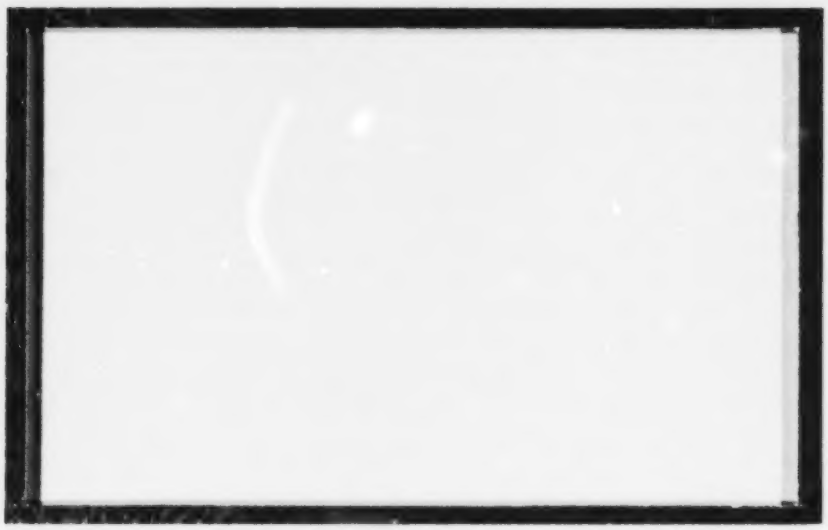

CENTRE DE RECHERCHE SUR LES TRANSPORTS

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A STUDY OF AGGREGATE BI-MODAL URBAN TRAVEL SUPPLY, DEMAND AND NETWORK BEHAVIOR USING SIMULTANEOUS EQUATIONS WITH AUTOREGRESSIVE RESIDUALS

by

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This research was supported by the Research and Development Centre of Transport Canada and by the F.C.A.C. program of the *Ministère de l'éducation du Québec*. Preliminary results were presented at International Symposium on Travel Supply Models, Montreal 1977. Final results were presented at the Workshop in Analytical Urban Economics, Queen's University, June 1978.

Queen's University

Discussion Paper # 290 - Institute for Economic Research

Université de Montréal

Publication # 88 - Centre de recherche sur les transports
Cahier # 7713 - Département de sciences économiques

Novembre 1977 - Révisé en août 1978

Dépôt légal 4e trimestre 1978

ABSTRACT

This paper emphasizes the existence of a difference among demand functions, which describe how consumers react, supply functions, which analyze the behavior of suppliers, and cost functions, which specify how prices and levels of service on a link or in a network vary with vehicle flows, ridership flows and other factors such as technology. Four aggregate demand, supply and cost models are formulated: each one regroups a subset of demand, supply and cost functions for two modes in Montreal. A significant part of the analysis pertains to the study of a regulated transit supplier. The parameters of all models are estimated by at least two of four limited-information and full-information estimation techniques. All of these procedures are designed to take proper account of equation-specific autocorrelation schemes of the residuals: (i) LSGAU, a least-squares generalized autoregressive estimator; (ii) SSGAU, a two-stage least squares generalized autoregressive estimator; (iii) ISUGAU, an iterated version of Parks' seemingly unrelated procedure generalized to multiple autocorrelation cases; and (iv) IFIVER, an iterated version of Fair's full-information instrumental variables efficient estimator. A sample comparison of results obtained by IFIVER and other full-information estimators is also provided: the latter are MIFIDA, a modified iterated version of Dhrymes' full-information dynamic autoregressive estimator, and FIML, the maximum likelihood estimator. It is shown that removing simultaneous equations biases has a significant impact on demand, supply and congestion function estimates: notably, it modifies the relative importance of waiting time and in-vehicle time elasticities of demand and the measures of the value of time associated with them.

RÉSUMÉ

Nous utilisons dans cet article une distinction entre les fonctions de demande, d'offre et de coût sur un lien ou dans un réseau de transport. Nous formulons quatre modèles qui mettent en lumière l'un ou l'autre des sous-ensembles de fonctions agrégées de demande, d'offre et de coût associées à deux modes de transport urbain; nous étudions de manière particulière le fonctionnement d'un monopole public réglementé qui offre ses services de transport en commun et dont les véhicules interagissent avec les automobiles sur le réseau routier urbain. Pour estimer les paramètres de nos quatre modèles, nous utilisons plusieurs techniques à information limitée ou complète qui sont toutes adaptées au problème de l'autocorrélation des erreurs résiduelles et utilisables dans les cas de systèmes d'équations simultanées non linéaires de grande taille. Les données utilisées pour les deux modes de transport considérés proviennent de la région montréalaise.

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1. INTRODUCTION: THE PROBLEM

The problem to be addressed in this paper is that of the formulation and estimation of the parameters of travel demand, supply and cost functions in an urban context. Consider, for expository purposes, a network consisting of a single link of length ℓ between points A and B. In the short-run, the physical infrastructure is given and the following system is of interest:

$$y_i^d = f_i^d\,(\{y_{km}^c\}\ ,\ X_i) + u_i\,, \qquad k=1,\ldots,K;\ m,i=1,2, \tag{1}$$

$$y_j^s = f_j^s\,([y_{kj}^c]\ ,\ y_j^d\ ,\ X_j) + u_j\,, \qquad k=1,\ldots,K;\ j=1,2, \tag{2}$$

$$y_{km}^c = f_{km}^c(\{y_{k'm}^c\},[y_i^d],[y_j^s],X_{km}) + u_{km}\,, \qquad k=1,\ldots,K;\ k\neq k';\ i,j,m=1,2. \tag{3}$$

where {} and [] respectively denote matrices and vectors of variables and

y_i^d = seat-mileage demanded on the i^{th} mode,

y_j^s = seat-mileage supplied by the j^{th} mode,

y_{km}^c = k^{th} price, service or comfort level of the m^{th} mode,

u_i, u_j, u_{km} = random terms of unspecified distribution.

We assume that the sets of independent variables X_i, X_j and X_{km} and of endogeneous variables contain enough restrictions to make all equations identifiable and that, for non stochastic equations or identities, the appropriate error term is zero. There are implicit time period subscripts on all variables and error terms.

1.1. Demand, supply and cost functions

Equations of type (1) are designed to answer questions of the form: how many people would like to travel by a certain mode if travel conditions

(y^c_{km}) and exogeneously determined factors (X_i) were at a certain level? Demand elasticities derived from estimating the parameters of equations of that type are of particular interest for policy purposes.

Equations of type (2) are of the form: how is the capacity (y^s_j) which producers are willing to supply determined? Price elasticities of supply derived from such equations are of particular interest for policy purposes because they summarize producers' reactions to market prices.

Equations of type (3) are of the form: how do ridership levels (y^d_i), the number of vehicles on the road [$f(y^s_j)$], physical conditions, management and other factors in X_{km} determine service and comfort levels (y^c_{km}) and prices? The decreasing returns associated with increased output levels expressed for instance in congestion functions are among the important pieces of information derived from such functions.

Without making an exhaustive survey of the literature, it is reasonable to note that there has traditionally been more interest in demand-cost ((1) , (3)) systems (henceforth D-C) than in demand-supply ((1) , (2)) systems (henceforth D-S) or in demand-supply-cost ((1) , (2) , (3)) systems (henceforth D-S-C). There has also been more interest in single-equation estimation of D, S and C functions than in joint estimation despite the possible gains from using full-information methods where the variables which are crucial for policy purposes, the price and service level demand elasticities, are precisely those which are endogeneous to the D-S-C system and which are the most likely to introduce simultaneous equations biases in demand parameter estimates.

D-S systems for the transit mode have been formulated and estimated by Nelson (1972), Veatch (1973) and Frankena (1976). Morlock (1976) has developed a spatial transit S function. Sherret (1971) has estimated bi-modal (auto vs rail; auto vs transit) D-C systems; Loose and Roueche (1977) have estimated an aggregate D-C system for a single mode. Most of these authors have used two-stage least-squares but none have made use of the possible correlation among u_i, u_j and u_{km} or used full-information estimation techniques. Numerous authors have estimated D and C functions separately at various levels of aggregation.

1.2. Market and network or link equilibrium

A basic difference between D-C systems and D-S-C systems is that the latter make the suppliers' decisions on the number of vehicles to be put in service endogeneous to the system while the former centre on the simultaneous determination of demands and service (or cost) levels under the assumption of exogeneously determined total numbers of supplied vehicles. *They neglect the suppliers' own equilibrium problem.* Indeed, within the simplified D-S-C framework defined by (1), (2) and (3), it is possible to distinguish between a number of notions of equilibrium. Market equilibrium requires that

$$y_i^d = y_i^s , \tag{4}$$

or the weaker condition

$$y_i^d \leq y_i^s . \tag{4'}$$

Nothing guarantees that it will be worthwhile for a producer to provide adequate service in every conceivable situation, or that (4') always holds.

Car trip equilibrium problems typically consist of combining a reduced form

car trip equation constructed from car supply and demand functions with network (or here, strictly speaking, link) cost functions, as in the DS-C set

$$y_i^{rf} = f_i^{rf}(\{y_{ki}^c\}, X_i, X_j) + u_i^{rf} ,$$
$$y_{ki}^c = f_{ki}^c(y_i^{rf}, X_{ki}) + u_{ki}, \qquad (5)$$

where i denotes the car, k typically denotes travel time and a link equilibrium is a set of values $\{{}^*y_i^{rf}, {}^*y_{ki}^c\}$ which solve the reduced form and the link cost functions simultaneously. For the transit mode, structural demand functions for transit trips are typically used in conjunction with flat cost functions. Florian (1977) and Florian et al. (1977) have recently integrated a DS-C spatial system for the car mode and a D-C spatial system for the transit mode. More complex notions of market and network equilibrium than those just used can be found in Gaudry (1976, 1978b).

1.3. Approach to be used

We will formulate D, S and C functions and estimate their parameters both by limited and by full-information techniques. We shall consider four distinct models. Model I is an S-C transit model providing a reasonably detailed formulation of S and C structural equations for a large and reasonably typical urban transit authority. Model II is a D-SC-C transit model to analyze the joint determination of supply,cost and demand relationships. One of the functions used in that model is called a SC function because it consists of a combination of S and C structural functions into a reduced form function effected in order to decrease the number of equations to be considered simultaneously. Model III is a D-C car trip model. Model IV combines the second and third models into a bi-modal D-SC-C formulation.

The purpose of our analysis is to study the workings of a regulated transit

supplier and to measure with sophisticated techniques both the impact of that supplier's actions on the demand for public and private transportation and the way in which demand feeds back on the supplier's behavior; we will also analyze the reciprocal determination of consumer/producer behavior and of travel costs on an aggregated network. The use of simultaneous equations estimation techniques is required if one is to estimate the parameters of structural demand, supply and cost functions without the presence of simultaneous equations biases or, stated otherwise, without interference among relationships. With an aggregate model, interference of this sort occurs, for instance in a *demand* equation, if producers increase service frequency in anticipation of increased demand: if that is the case and one forgets it in the estimation of frequency of service travel demand elasticities, the frequency of service variable will be positively correlated with ridership levels even if consumers are indifferent to frequency of service; similarly, it could be found that increased speeds are associated with lower ridership levels because higher ridership slows vehicles down. Removing those biases distinguishes the extent to which apparent correlations reflect how consumers are truly reacting to service levels from the extent to which they reflect the behavior of suppliers or of the network costs. Analogous problems can also occur in the estimation of *supply* and *costs* functions. Trying to remove such biases is justified in three of our four aggregate time-series models. Of course simultaneous equations biases can also arise from the correlation among stochastic terms of demand, supply and cost equations in both aggregate and disaggregate models and, in that sense, the use of simultaneous equation estimation techniques should not be thought of as useful only in aggregate contexts.

2. SPECIFYING THE FOUR MODELS

2.1. Model I: a transit supply-cost (S-C) system

Consider the following equations which pertain to a public transit authority and where time subscripts denoting the period over which these relationships are defined are not written:

$$S \begin{cases} UVEH^S = f_1 (\tilde{TR}, \tilde{UVC}, \tilde{FC}, W) + u_1, & (6) \\ SM^S = UVEH^S . V . SC , & (7) \end{cases}$$

$$C \begin{cases} HW = \dfrac{CL . H . 60}{UVEH^S . V} & (8) \\ V = f_2 (UVE^S, AVE, SM^d, VETCH, W) + u_2 , & (9) \\ F = \bar{F}, & (10) \end{cases}$$

$$S \text{ or } C \begin{cases} CL = f_3 (AR, MTCH) + u_3 , & (11) \end{cases}$$

where $UVEH^S$ = unit vehicle hours supplied by transit authority,
$\tilde{TR}$ = expected total revenues of transit authority,
$\tilde{UVC}$ = expected unit variable costs,
$\tilde{FC}$ = expected fixed costs,
W = weather,
SM^S = seat-mileage supplied,
V = velocity of transit vehicles,
SC = seat capacity of identical transit vehicles,
HW = headway between transit vehicles in minutes,
CL = circuit length of vehicle routes,
H = total number of hours of operation,
UVE^S = number of vehicle units put in service,
AVE = number of automobiles competing for use of roads,
SM^d = seat demand by transit riders,
$VETCH$ = transit vehicle speed technology,
F = exogeneously determined transit fares,
AR = area served by transit authority,
$MTCH$ = modal technology (tramway, trolleybus, bus, metro).

Let us explain the meaning of the three groups of equations just listed.

2.1.1 Supplying seat-mileage under a zero profit constraint

Equations (6) and (7) are an explicitation of Equation (2). The transit authority of interest to us, the Montreal Urban Community Transit Commission (henceforth MUCTC) is a regulated public monopoly whose objective is to make a zero profit or balance total revenues TR and total costs TC, taking one year with the next. In the short run — on a monthly or yearly basis — it considers its fare as fixed and it meets its budget constraint by changing the number of vehicle unit hours (tramways, trolleybuses, buses, subway trains) supplied per time period. We have explained elsewhere (Gaudry, 1973) that the total number of unit hours (most variable costs can be associated to unit vehicle hours) is basically determined at budget time by the Board of Directors and is not normally subjected to short-run revisions.

More precisely, a fair representation of the MUCTC's intended behavior at budget time can be expressed by the following equation

$$\left.\begin{aligned} &\tilde{TC} - \tilde{TR} = 0 \\ &(\tilde{VC} + \tilde{FC}) - \tilde{TR} = 0 \\ &UVEH^S \cdot \tilde{UVC} + \tilde{FC} - \tilde{TR} = 0 \\ &UVEH^S = \frac{\tilde{TR} - \tilde{FC}}{\tilde{UVC}} = \frac{\tilde{PR} + \tilde{SUB} - \tilde{FC}}{\tilde{UVC}} \end{aligned}\right\}, \qquad (12)$$

where

$\tilde{TC}$ = expected total costs,
$\tilde{VC}$ = expected variable costs,
$\tilde{FC}$ = expected fixed costs,
$\tilde{UVC}$ = expected unit variable cost of vehicle hours,
$\tilde{PR}$ = expected passengers revenues,
$\tilde{SUB}$ = expected subsidies from local or higher levels of government.

The MUCTC's actual behavior during any month t can be described by an explicated[@] and augmented version of the last form of (12):

$$UVEH_t^s = \left\{ \frac{\left[F_t.(SM_{t-12} + NRA_t).HET_t\right] + S\tilde{U}B_{t-b} - F\tilde{C}_t}{(HWRD_t.MTCH_t) + OVIC_t} \right\} . W_t + u_{1_t} , \qquad (13)$$

where previously undefined symbols are, apart from budget time subscript $b \geq 3$,

NRA_t = non recurrent activities such as the world fair Expo'67 and unexpected sport or cultural events of large size for which additional service will be provided on regular lines;

HET_t = an heterogeneity factor based on the number of workdays, Saturdays and Sundays or holidays; this factor reflects the fact that the *observed* demand period of the previous year may not be homogeneous with the current period;

$HWRD_t$ = the hourly wage rate of vehicle drivers, a crucial determinant of variable costs; the amount used per vehicle hour differs among sub-modes (buses, tramways, subway trains, trolleybuses); it is assumed to be exogeneous to the model despite the possibility that it may be related to the level of the fare;

$OVIC_t$ = other variable input unit costs.

The specification (13) assumes that the weather, W_t, has a proportional impact on the intended supply of vehicle hours. The weather causes differences between the intended and actual levels of vehicle-hours supplied because it affects the number of drivers who show up for work and, in large storms, may make the provision of service altogether undesirable. The term u_{1_t} represents other unspecified causes of disruption or errors of observation on the dependent variable. Although (13) is a nonlinear equation, we will use a linear form[@@]. The reason for this is not simply that a linear

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[@] The basic format of this equation was suggested by D.N. Dewees of the University of Toronto.

[@@] A few attempts made with partially nonlinear forms and with built-in expectations of price elasticity of demand did not produce results significantly different from those reported later.

approximation can be justified when the range of observations is not very large but that nonlinearities increase the computational difficulties posed by the simultaneous estimation of sets of interdependent equations. A crucial feature of a linear form of (6) or (13) is, for our purposes, the fact that the vehicle-hour supply adjustment to changes in demand does not primarily depend on *contemporaneous* demand variations and, as a first approximation, may justify considering the service level factor wait-time as exogeneous to transit demand equations. This assumption will later be relaxed to allow for a degree of unintended simultaneity (through random terms correlated across supply and demand equations) in the determination of demand and wait-time.

Once the number of unit hours is determined, the supplied capacity SM^S will depend on actual speeds and the seat-capacity of vehicles, as the identity (7) makes clear.

2.1.2 Direct and indirect determination of consumers' costs

It is the policy of the MUCTC to extend the circuit routes to new developments in its territory in such a way as to insure that no origin or destination be further than one quarter of a mile from a circuit. This means that the structure of access times to stops is approximately constant over time and that the length of circuit routes CL is function of the urbanized area to be served, AR, given the modal technology structure MTCH, as shown in (11). The decision on CL is analogous to a decision on infrastructure by a government and can be classified either as a supply decision or as a decision on one element of network cost for the consumer: the density of lines determines

access times to stops or stations. In our case, its interpretation makes no difference. An error term is useful in (11) because street repairs, new one-ways, etc., cannot reasonably be accounted for explicitly in an aggregate model. Access *time* to the network is the result of a direct decision on the MUCTC's part.

Decisions on circuit length, CL, and unit hours, UVEH, having been taken, the actual average headway, HW, (and the implied average waiting time) "trickles down" and depends on realized speeds, as shown in (8). *Wait-time* for transit vehicles is therefore indirectly determined by the MUCTC because it depends on factors which are not under the firm's control.

The speed of transit vehicles depends on the underlying modal technology (VETCH) which is exogeneous in this model, on the number of transit vehicles (UVE^S)actually in service, on the number of competing cars (AVE), on the weather (W) and on the actual demand for transit (SM^d): in Montreal the commercial speed of subway trains is constant (there is no congestion, even at peak time) but higher usage of the other submodes (tramways, trolley-buses, buses) slows them down. *Travel time*, or in-vehicle time, is then also only partially under MUCTC control. Although micro-economic formulations of congestion functions, well surveyed in Branston (1975), are nonlinear, we shall use a linear form for the reasons stated in the previous sub-section.

All of the above relate to the determination of consumers' travel time costs. In a long-run model, the money cost of trips should be endogeneous to the model. In the short-run model considered here, the fare is, in (10),

assumed to change exogeneously and infrequently: transit authorities do not like to change the fare often and will increase it as a measure of last resort or if that is the only way to maintain the basic or minimum level of service on relatively unprofitable lines.

2.1.3 Other particulars

In this description of the regulated monopolistic supply process (6) - (11), three relationships are identities; in estimating the remaining stochastic equations (6), (9) and (11), we will include the classes of variables listed above except for UVE^S, the actual number of vehicles put in service. The reason for this omission is the absence of reliable data on this variable; with the monthly averages which we are using, it might not have been the ideal variable for our purposes, even if the data had been available. In any event, the amount of self produced congestion in the transit network as a whole is, the MUCTC believes, "insignificant"; to that extent, our estimates of the resulting parameters of (9) should not be very biased by the absence of that variable. We will be particularly interested in the price elasticity of supply derived from (6). As formulated, the set of these three equations contains no variables endogeneous to *that* system. We will therefore use single equation estimation technique LSGAU and a technique which takes into account contemporaneous covariances among u_1, u_2 and u_3, ISUGAU: both techniques will be explained later.

2.2 Model II: a transit demand-supply/cost-cost (D-SC-C) system

Consider the following "direct" transit demand equations for period t for adults (subscript 1) and children (subscript 2):

$$SM^d_{1t} = d_1 (TW_t, TT_t, T_t, P_{1t}, W_t, A_{1t}, Y_t) + u_{1t} \quad , \tag{14}$$

$$SM^d_{2t} = d_2 (TW_t, TT_t, T_t, P_{2t}, W_t, A_{2t}, Y_t) + u_{2t} \quad , \tag{15}$$

where

SM^d = seat demand,

TW = transit wait-time,

TT = transit in-vehicle time,

T = in-vehicle travel time by car

P = prices of transit, car and other goods and services,

W = vector of weather variables,

A = vector of activity levels,

Y = income.

Equations (14) and (15) are an updated version of those found in Gaudry (1978a) and used by the MUCTC to obtain monthly passenger forecasts.

For the supply/cost side, write structural S-equation (6) with all variables from (13) and combine it with structural C-equation (8) to obtain, after a transformation of headway (HW) into wait-time (TW) and of speed (V) into travel time (TT), the following modified reduced-form or combined SC function:

$$TW_t = sc_3 (TT_t, SM_{t-12}, OTHER_t) + u_{3t} \; . \tag{16}$$

Transform the structural congestion or cost function (9) into

$$TT_t = c_4\ (SM^d_{1t},\ SM^d_{2t},\ AVE_t,\ VETCH_t,\ W_t) + u_{4t} \tag{17}$$

which differs from (9) in that travel time is used as dependent variable instead of speed and that, among its arguments, trip demand by type of person is used instead of total trip demand; again, the absence of data on the number of vehicles in service, UVE^s, forces us to exclude that variable from aggregate network congestion function (17).

The system under consideration can also be written as

$$\left.\begin{aligned}
SM^d_{1t} &= d_1(TW_t,\ TT_t,\ X_{1t}) + u_{1t}\ ,\\
SM^d_{2t} &= d_2(TW_t,\ TT_t,\ X_{2t}) + u_{2t}\ ,\\
TW_t &= sc_3((SM_1 + SM_2)_{t-12},\ TT_t,\ X_{3t}) + u_{3t}\ ,\\
TT_t &= c_4(SM^d_{1t},\ SM^d_{2t},\ X_{4t}) + u_{4t}\ ,\\
SM^s_t &= s(TW_t,\ X_{5t})\ ,\\
SM^s_t &\geq SM^d_{1t} + SM^d_{2t}\ ,
\end{aligned}\right\} \tag{18}$$

where $X_1, \ldots, X_5$ are sets of distinct exogeneous explanatory variables. The first four equations are stochastic, the fifth one is an identity and the

last one an excess supply condition required for a market equilibrium to be observed. We have explained elsewhere (Gaudry 1975) why this last condition is reasonable for our sample period. The parameters of the first four equations of D-SC-C system (18) will be estimated by simultaneous equations estimation technique IFIVER which takes into account both the presence of current or lagged endogeneous variables in each equation and contemporaneous correlation among the error terms of equations. This procedure will allow us to remove the simultaneous equations bias both from demand equation estimates and from supply-cost function estimates. We will be particularly interested in the impact of these procedures on price and service level elasticities of demand and on the price elasticity of service. The results obtained in this way will be compared to results obtained by the limited information equation estimation technique LSGAU which assumes that error terms across equations are not correlated and by the ISUGAU technique which assumes that they are.

2.3. Model III: a car demand-cost (D-C) system

We shall consider the following system which consists of a demand and of a cost or congestion equation:

$$\left.\begin{aligned} AVE_t^d &= d\,(\,T_t\,,\,X_{6t}) + u_{1t}\,, \\ T_t &= c\,(AVE_t^d,\,X_{7t}) + u_{2t}\,. \end{aligned}\right\} \qquad (19)$$

In the first equation, the set of exogeneous variables X_{6t} comprises car and transit trip prices,

transit service levels, activity levels and weather factors. The in-vehicle time by car (T) explained by the second equation depends on car trip demand and on a set of factors in X_{7t} which comprises the variable TOWN which represents the development of one way streets by the traffic department. The absence of data on many pertinent variables and the poor quality of our data on T argue against the use of a more sophisticated model: the cost of gas influences driving speed behavior and has been included as an explanatory variable of T. It would in principle be possible to add to (19) another cost function to explain the cost of auto trips, FACAR, in terms of speed, taxation levels and other regulatory measures (tolls, etc.) as follows

FACAR = c (cost of gas, realized car speed, taxes, tolls, etc.), (20)

but the absence of good aggregate data on these variables prevents us from extending the simultaneous equations system (19). It should also be possible to formulate identifiable car trips supply functions but the absence of data on the number of licensed drivers and other variables which would make it possible to identify such supply functions compels us not to use a supply function in this model. Results will be obtained with the single equation estimation techniques LSGAU and SSGAU, the latter of which takes into account the presence of endogeneous variables as regressors. Full information techniques will not be used because poor observations on travel time by car (T) would yield biased estimates of error term u_{2t} in (19).

2.4. Model IV: a bi-modal demand-supply/cost-cost (D-SC-C) system

Models I, II and III describe travel modes separately. It is of interest to consider them jointly. In order not to have too large a system, we will aggregate the adult and children transit demand functions and exclude

from the simultaneous framework the car in-vehicle time cost function used in (19). Our system will consist of

$$\left.\begin{aligned} SM^{d}_{1+2_t} &= d_1\,(TW_t,\ TT_t,\ X_{1t},\ X_{2t}) + u_{1t}, \\ AVE^{d}_{t} &= d_2\,(TW_t,\ TT_t,\ X_{6t}) + u_{2t}, \\ TW_t &= sc_3\,(SM_{1+2_{t-12}},\ TT_t,\ X_{3t}) + u_{3t}, \\ TT_t &= c_4\,(SM^{d}_{1+2_t},\ X_{4t}) + u_{4t}, \end{aligned}\right\} \qquad (21)$$

and we will assume that the last two relationships used previously in (18) hold. In this model, we will proceed as for Model II, using the single-equation procedure LSGAU, the full-information estimator IFIVER and the grouped equation procedure ISUGAU. The latter takes contemporaneous correlations across equations into account but assumes that all explanatory variables of the system are exogeneous.

3. SIX ESTIMATION TECHNIQUES

We will assume that each of the equations to be considered in each model is linear in variables or can be approximated by a linear form in the relevant range of the observations. This assumption has been shown to be adequate for the two demand equations of model II (Gaudry and Wills, 1977) but has not been verified for the other equations of interest in this study.

We envisaged estimating with Box-Cox (1964) transformations the functional form or each equation at the same time as the other parameters of the models by extending Spitzer's (1977) approach to account for multiple autocorrelation of

the residuals. One reason for deciding otherwise was the enormously increased computational burden which such a course of action would have implied. Another was the basic difficulty, in existing procedures, of distinguishing between the extent to which estimated functional form parameters compensate for heteroskedasticity of the residuals and the extent to which they reflect nonlinearities really present in the model: although some research is currently done on this problem (e.g. Gaudry and Dagenais, 1978), the estimation of the form of simultaneous equations with heteroskedastic autoregressive residuals seemed premature.

We consequently centered our work on refining existing estimation procedures for linear systems in such a way that, after taking serial correlation into account in a specific way in each equation, "white noise" residuals are obtained, a result we achieved in all equations except the automobile congestion (T) equation shown in Table 12. The adopted refinements had on the results an impact which suggested that generalizations of the procedures to include equation-specific multiple autocorrelation schemes notably increased the efficiency of the procedures for our sample size of 181 consecutive monthly observations; the less efficient results obtained without these generalizations will not be reported because of the length of this paper. As practical modifications of the procedures are not always explicated in the original theoretical articles, or even in textbooks (e.g. Granger and Newbold, 1977), we shall presently provide a brief summary. Some concepts needed in the following

section will be defined in this section but the reader in a hurry can proceed directly to the reported results.

Specifically, we will consider relationships of the standard form

$$y_{i_t} = Y_{i_t}\gamma_i + X_{i_t}\beta_i + u_{i_t}, \qquad \begin{matrix} i=1,\ldots,p, \\ t=1,\ldots,n, \end{matrix} \tag{22}$$

$$u_{i_t} = \sum_{\ell=1}^{m} \rho_{i_\ell} u_{i_{t-\ell}} + e_{i_t}, \tag{23}$$

in which

- y_{i_t} denotes the t^{th} observation on the dependent variable y_i,
- Y_{i_t} denotes the t^{th} observation on a row vector of endogeneous variables other than y_{i_t},
- X_{i_t} denotes the t^{th} observation on a row vector of exogeneous variables,
- γ_i and β_i are column vectors of coefficients,
- u_{i_t} denotes error terms determined by a specific m^{th}-order auto-regressive process where e_{i_t}'s are independent and identically distributed with zero mean and variance-covariance matrix $\sigma_{ii}^2 I$, and where each vector $\rho_i = (\rho_{i_1}, \ldots, \rho_{i_\ell}, \ldots, \rho_{i_m})$ of auto-regressive parameters satisfies standard stationarity assumptions.

Combining (22) and (23), each relationship considered may be written

$$(y_{i_t} - \sum_{\ell=1}^{m} \rho_{i_\ell} y_{i_{t-\ell}}) = (Y_{i_t} - \sum_{\ell=1}^{m} \rho_{i_\ell} Y_{i_{t-\ell}})\gamma_i + (X_{i_t} - \sum_{\ell=1}^{m} \rho_{i_\ell} X_{i_{t-\ell}})\beta_i + e_{i_t}, \tag{24}$$

$$i = 1,\ldots,p, \quad t = m+1,\ldots,n.$$

We are now in a position to outline the estimation procedures to be used.

3.1 The least-squares generalized autoregressive technique LSGAU

Assume that the i^{th} Equation (24) contains no endogeneous (current or lagged) variables on the right hand side. Specify the equation-specific autoregressive scheme by examining, in the spirit and Box and Jenkins (1970), the autocorrelation and partial autocorrelation functions of the unbiased estimates of the residuals $\hat{u}_{i_t}$ obtained from applying least-squares to (22); then solve the particular Equation (24) so specified by the Cochrane-Orcutt (1949) iterative technique which is a maximum likelihood procedure of somewhat lower efficiency than a fuller maximum likelihood procedure which does not ignore a number of observations equal to the highest order of autocorrelation considered (Beach and MacKinnon, 1978). LSGAU is identical to the single equation estimation procedure S used in Gaudry (1975, 1978a).

3.2 The two-stage-least-squares generalized autoregressive technique SSGAU

Assume that the i^{th} Equation (24) contains endogeneous variables on the right-hand side. Extending Fair's (1970) procedure to cases of multiple autocorrelation, choose a set of instrumental variables which are uncorrelated with e_{i_t} and which at least include $y_{i_{t-\ell}}$, $Y_{i_{t-\ell}}$, X_{i_t} and $X_{i_{t-\ell}}$; regress each of the variables in Y_{i_t} on this set and calculate the predicted values of Y_{i_t} (denoted $\hat{Y}_{i_t}$) from these regressions. Substitute $\hat{Y}_{i_t}$ for Y_{i_t} in (24) and solve, as in LSGAU, the following non linear equation:

$$\left(y_{i_t} - \sum_{\ell=1}^{m} \rho_{i_\ell} y_{i_{t-\ell}}\right) = \left(\hat{Y}_{i_t} - \sum_{\ell=1}^{m} \rho_{i_\ell} Y_{i_{t-\ell}}\right) \gamma_i + \left(X_{i_t} - \sum_{\ell=1}^{m} \rho_i X_{i_{t-\ell}}\right) \beta_i + e_{i_t}, \qquad (25)$$

$$i = 1,\ldots,p,$$

$$t = m+1,\ldots,n.$$

This equation is the generalized autoregressive analog of the second stage of the two-stage-least-squares estimator — whence its name —. Neither LSGAU nor SSGAU take into account the contemporaneous correlation of error terms e_{i_t} across equations.

3.3 The iterated seemingly unrelated generalized autoregressive technique ISUGAU

Assume that none of the set of p equations considered in (22) contain endogeneous variables on the right-hand side. Then each equation can be written

$$y^*_{i_t} = X^*_{i_t}\beta_i + e_{i_t}, \qquad \begin{matrix} i = 1,\ldots,p, \\ t = m+1,\ldots,n, \end{matrix} \tag{26}$$

where starred values respectively denote the contents of the first and third parentheses of (24). Then write the system considered as

$$\begin{bmatrix} y^*_{1_t} \\ \cdot \\ \cdot \\ \cdot \\ y^*_{p_t} \end{bmatrix} = \begin{bmatrix} X^*_{1_t} & & \bigcirc \\ & \cdot & \\ & & \cdot \\ \bigcirc & & X^*_{p_t} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \cdot \\ \cdot \\ \cdot \\ \beta_p \end{bmatrix} + \begin{bmatrix} e_{1_t} \\ \cdot \\ \cdot \\ \cdot \\ e_{p_t} \end{bmatrix}, \quad t = m+1,\ldots,n, \tag{27}$$

or as

$$y^*_t = X^*_t\beta + e_t, \qquad t = m+1,\ldots,n. \tag{28}$$

Assume further that error terms are contemporaneously correlated across equations or that

$$\Sigma = E(e_{i_t} e_{j_s}) = \begin{cases} \sigma_{ij} \text{ for } i,j = 1,\ldots,p \text{ and } t = s, \\ \\ 0 \quad \text{for } i,j = 1,\ldots,p \text{ and } t \neq s. \end{cases} \tag{29}$$

To obtain a consistent estimate of Σ, use the consistent estimate of e_{i_t} obtained by applying least-squares on (23) for each equation taken separately, the ρ_i vectors (24) being identified as in procedure LSGAU. Writing

$\Sigma = \Sigma_c \otimes I$, where $\otimes$ denotes Kronecker multiplication of matrices, and I is an $\underline{n} \times \underline{n}$ identity matrix, each element of Σ_c may be computed thus

$$\hat{\Sigma}_c = \{\hat{\sigma}_{ij}\} = \frac{\hat{e}_i \hat{e}_j}{(n-m-k_i)^{\frac{1}{2}} (n-m-k_j)^{\frac{1}{2}}} \quad , \tag{30}$$

where k_i and k_j respectively denote the number of explanatory variables in the i^{th} and j^{th} equations. Then use this estimate $\hat{\Sigma}_c$ to obtain the generalized or Aitken estimator of the β parameters and of their standard errors in the transformed system

$$\hat{\beta} = (X^{*\prime} \Sigma^{-1} X^*)^{-1} X^{*\prime} \Sigma^{-1} y^* \quad , \tag{31}$$

where t subscripts have been dropped to indicate the replacement of individual observations by column vectors of observations. This amounts to a generalization of Parks' (1967) procedure to higher orders of autocorrelation than tne first. Then iterate on the $\hat{\rho}_i$, $\hat{\Sigma}$ and $\hat{\beta}$ until convergence is reached in order to obtain a maximum likelihood estimate (Chow and Fair, 1973). ISUGAU is identical to the grouped-equation procedure G used in Gaudry (1978a).

3.4 The iterated full-information instrumental variables efficient technique IFIVER

Rewrite the structural equations in (22) in the form

$$y_{i_t} = Z_{i_t} \delta_i + u_{i_t} \quad , \qquad \begin{matrix} i = 1,\ldots,p, \\ t = 1,\ldots,n, \end{matrix} \tag{32}$$

where $Z_{i_t} = [Y_{i_t} \; X_{i_t}]$, $\delta_i = \begin{bmatrix} \gamma_i \\ \beta_i \end{bmatrix}$,

and combine them to yield

$$y_t = Z_t \, \delta + u_t \, , \tag{33}$$

where

$$y_t = \begin{bmatrix} y_{1_t} \\ \vdots \\ y_{p_t} \end{bmatrix}, Z_t = \begin{bmatrix} Z_{1_t} & & \bigcirc \\ & \ddots & \\ \bigcirc & & Z_{p_t} \end{bmatrix}, \delta = \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_p \end{bmatrix}, u_t = \begin{bmatrix} u_{1_t} \\ \vdots \\ u_{p_t} \end{bmatrix} .$$

Define a set of m matrices of autoregressive parameters $R^{(1)} \ldots R^{(\ell)} \ldots R^{(m)}$ of dimension $\underline{p} \times \underline{p}$. To implement the estimation procedure of interest here, it is necessary to transform (33) so that the error term is e_t rather than u_t. This transformation is

$$\begin{aligned} y_t - [(R^{(1)} \otimes I) y_{t-1} + \ldots + (R^{(m)} \otimes I) y_{t-m}] = \\ \{Z_t - [(R^{(1)} \otimes I) \, Z_{t-1} + \ldots + (R^{(m)} \otimes I) \, Z_{t-m}]\} \, \delta + e_t \quad , \end{aligned} \tag{34}$$

where I is the identity matrix of order $\underline{1} \times \underline{1}$ here and $\underline{n} \times \underline{n}$ when t subscripts are developped. Equation (34) is simply the analog of (28) and can be written in the form

$$y_t^* = Z_t^* \, \delta + e_t \, , \qquad t = m+1, \ldots, n \; . \tag{35}$$

It is convenient to use still another way of writing the system of structural Equations (22). Let us use

$$Y_t \, \Gamma + X_t \, B = U_t \; , \tag{36}$$

where Y_t is the t^{th} observation on a $\underline{1} \times \underline{p}$ vector of endogeneous variables,

X_t is the t^{th} observation on a $\underline{1} \times \underline{k}$ vector of predetermined variables,

Γ and B are $\underline{p} \times \underline{p}$ and $\underline{k} \times \underline{p}$ matrices of coefficients,

U_t is the t^{th} $\underline{1} \times \underline{p}$ vector of error terms and is assumed to follow an m^{th} order autoregressive process:

$$U_t = U_{t-1} R^{(1)} + \ldots + U_{t-\ell} R^{(\ell)} + \ldots + U_{t-m} R^{(m)} + E_t. \qquad (37)$$

Combining (36) and (37) yields

$$Y_t\Gamma + X_tB = Y_{t-1}\Gamma R^{(1)} + X_{t-1} B R^{(1)} + \ldots + Y_{t-m}\Gamma R^{(m)} + X_{t-m} B R^{(m)} + E_t. \qquad (38)$$

From (38), the reduced form for Y_t is

$$Y_t = - X_tB\Gamma^{-1} + Y_{t-1}\Gamma R^{(1)} \Gamma^{-1} + X_{t-1} B R^{(1)} \Gamma^{-1} + \ldots + Y_{t-m} \Gamma R^{(m)} \Gamma^{-1} + X_{t-m} B R^{(m)} \Gamma^{-1} + E_t\Gamma^{-1} , \qquad (39)$$

or

$$Y_t = Q_t \Pi + V_t, \qquad (40)$$

where

$V_t = E_t\Gamma^{-1}$, $Q_t = [X_t\ Y_{t-1}\ X_{t-1} \ \ldots\ Y_{t-m}\ X_{t-m}]$ and Π is partitioned according to Q.

Now drop t subscripts to indicate the replacement of individual observations by vectors of observations. We are then in a position to describe the FIVER estimator and its iterated version IFIVER. Proceed as follows:

(i) Assume that initial consistent estimates of Γ, B and the $R^{(\ell)}$ matrices are available so that a consistent estimate of Π in (40) is available. This estimate can be used to generate, from (40), consistent predictions of the endogeneous variables in the model. We shall state later how this initial set of consistent estimates is obtained.

(ii) Let $\hat{Z}$ denote the matrix Z except for the replacement of the current endogeneous variables in Z by their predicted value from (40).

(iii) Let $\bar{y}^*$ and $\bar{Z}^*$ denote the matrices y* and Z* transformed with consistent estimates of $R^{(\ell)}$ matrices rather than with actual or true values of these matrices.

(iv) Let $\hat{\bar{Z}}^*$ denote the matrix $\bar{Z}^*$ except for the replacement of the current value of Z by $\hat{Z}$ as follows:

$$\hat{\bar{Z}}^* = \bar{Z} - [(\hat{R}^{(1)}{}' \otimes I)\, Z_{-1} + \ldots + (\hat{R}^{(m)}{}' \otimes I)\, Z_{-m}]. \tag{41}$$

(v) Then Fair's (1972) FIVER estimator is, using $\Sigma = \Sigma_c \otimes I$ from (29),

$$d = (\hat{\bar{Z}}^{*\prime}\, (\hat{\Sigma}_c^{-1} \otimes I)\, \bar{Z}^*)^{-1}\, \hat{\bar{Z}}^{*\prime}\, (\hat{\Sigma}_c^{-1} \otimes I)\, \bar{y}^*, \tag{42}$$

where $\hat{\Sigma}$ is a consistent estimate of Σ,

or, if we define $\bar{W} = (\hat{\Sigma}_c^{-1} \otimes I)\, \hat{\bar{Z}}^*$,

$$d = (\bar{W}'\, \bar{Z}^*)^{-1}\, \bar{W}'\, \bar{y}^* \tag{43}$$

which is a readily recognizable instrumental variables format.

Fair has shown that, in the case of known $R^{(\ell)}$ matrices, the FIVER estimation is asymptotically efficient. He has also shown that, for the case in which the $R^{(\ell)}$ matrices have to be estimated, IFIVER, based on iterating back and forth between estimates of δ and estimates of the $R^{(\ell)}$ matrices until convergence is reached, has the same asymptotic distribution as the full-information maximum likelihood estimator (FIML). This point has been restated by Dhrymes and Taylor (1976). In this case, we iterated as follows:

i) given the FIVER estimator $\hat{\delta}^{(k)}$, where the k^{th} iteration is k = 1, compute $\hat{U}$ in (36) from the subvectors of $\hat{\delta}^{(k)}$, namely $\hat{\gamma}^{(k)}$ and $\hat{\beta}^{(k)}$;

ii) estimate $\hat{R}^{(\ell)(k)}$ from (37);

iii) update reduced form values of $\bar{Y}$ from (40);

iv) obtain the IFIVER estimator $\hat{\delta}^{(k+1)}$ and cycle until the following convergence criterion is met:

$$\frac{\hat{\delta}^{(k+1)} - \hat{\delta}^{(k)}}{\hat{\sigma}_{\delta^{(k+1)}}} \leq \text{criterion value}, \tag{44}$$

where the denominator is taken from the standard error of $\hat{\delta}$ estimated by $(\overline{W}' \overline{Z}^*)^{-1}$.

The initial consistent estimates of Γ, B and the $R^{(\ell)}$ matrices used to obtain the first (k = 1) IFIVER iterate are calculated, following Fair, by:

i) treating all lagged endogeneous variables in (36) as current endogeneous variables using an instrumental variables estimator of Γ and B in (36), without taking correlation into account, for each equation taken separately;

ii) using these estimates to compute consistent estimates of U in (36) and, from those, consistent estimates of the $R^{(\ell)}$ in (37), taking each equation separately.

In all of our applications, we assumed that the $R^{(\ell)}$ matrices were diagonal, as is implicitly the case in (23).

3.5 The modified iterated full-information dynamic autoregressive technique MIFIDA[1]

The initial estimates of Γ, B and the $R^{(\ell)}$ matrices just described can be

1. Procedure MIFIDA generalizes to higher orders of autocorrelation than the first, and iterates on, procedure MFIDA which combines Fair's technique for obtaining initial or prior estimates of Γ and B from an instrumental variables estimator with the second step of Dhrymes' FIDA technique. MFIDA and MIFIDA were developed and programmed during the second half of 1975 and the first half of 1976 without knowledge of Dhrymes and Taylor's (1976) virtually identical "efficient two-step estimator". In their paper, Dhrymes and Taylor showed that their "efficient two-step procedure" (and by implication MFIDA and MIFIDA) is asymptotically equivalent to the full-information maximum likelihood estimator (FIML) and to the converging iterate of FIDA.

used as the initial step for Dhrymes' (1971) three stage least-squares-like procedure termed the full-information dynamic (FIDA) estimator which, in its iterated version when convergence is reached, satisfies asymptotically the same set of normal equations as the full-information maximum likelihood estimator and differs from it and from Fair's by the manner in which the jointly dependent variables are "purged" of their stochastic components, as Dhrymes and Erlat (1973) have shown.

Using the same initial estimates as those which permit to compute the first iteration of IFIVER, we can define MIFIDA as

$$d = (\bar{W}' \hat{\bar{Z}}^*)^{-1} \bar{W}' \bar{y}^* \ , \tag{45}$$

where the analogy with a three stage least squares estimator arises through the use of $\hat{\bar{Z}}^*$ in the parenthesis of (45), as compared to $\bar{Z}^*$ in FIVER above in (43). To obtain MIFIDA, we cycled as for IFIVER. One does not expect MIFIDA to be as efficient as IFIVER because of the information loss involved in using $\hat{\bar{Z}}^*$ instead of $\bar{Z}^*$, although that should not matter asymptotically.

3.6 The full information maximum likelihood technique FIML

If to the statistical specifications of our model (36) - (37) we add the assumption that the disturbances e_t are normally distributed, the log concentrated likelihood function can be written as in Chow (1964,1968):

$$L = -\frac{pn}{2} \ell n\, 2\pi - \frac{n}{2} \ell n \left|\frac{E'E}{n}\right| + \frac{n}{2} \ell n \left|\frac{\Gamma Y' Y \Gamma}{n}\right| \ , \tag{46}$$

where E is obtained from (38). Following Chow and Fair (1973), estimates of Γ, B and the $R^{(\ell)}$ can be obtained by repeated maximization with respect to Γ and B, for given values of the $R^{(\ell)}$, and with respect to

the $R^{(\ell)}$, for given values of Γ and B, until convergence is reached. This two-step iterative approach is adopted in the Chapman and Fair (1972) FIML program which we used with a small sample model in order to obtain an idea of the efficiency of IFIVER and MIFIDA procedures for our sample.

The FIML program and other programs used for this paper compute covariance matrices of the estimates of Γ and B (or of δ) under the assumption that the $R^{(\ell)}$ are known and compute the covariance matrices of the latter under the assumption that the former are known. The resulting values are, of course, underestimates of the actual covariance matrices.

3.7 A test comparison of FIML, IFIVER and MIFIDA results

We have performed sample calculations on a small model in order to compare FIML, IFIVER and MIFIDA results. The test model was

$$\left.\begin{aligned} SM^d_t &= \beta_{10} + \beta_{11}\, TW_t + \beta_{12}\, x_{2_t} + \beta_{13}\, x_{3_t} + \beta_{14}\, x_{4_t} + u_{1_t}, \\ TW_t &= \beta_{20} + \beta_{21}\, SM^d_{t-12} + \beta_{22}\, x_{5_t} + u_{2_t}, \\ u_{i_t} &= \rho_{i_1}\, u_{i_{t-1}} + \rho_{i_2}\, u_{i_{t-2}} + e_{i_t}, \quad \begin{matrix} i = 1, 2, \\ t = 1, \ldots, 181. \end{matrix} \end{aligned}\right\} \quad (47)$$

The first equation relates total transit demand (SM) to waiting time (TW) and travel time (x_4) by that mode, and to employment (x_2) and school (x_3) activities. In the second equation, transit waiting-time is explained by

lagged transit demand and the wage rate of vehicle drivers (x_5).

Both IFIVER and MIFIDA procedures were allowed 25 iterations, by which time at least seven digits of coefficients were identical to those of the 24^{th} iteration. Both procedures were used with two distinct sets of instruments at the initial stage. All results are reported in Table 1. We note the following:

i) the choice of initial instruments (predetermined variables in each case) did not influence the results at the 25^{th} iteration. Only the paths to these points differ, when IFIVER and MIFIDA listings are examined;

ii) both sets of results are extremely close to FIML results, but IFIVER results are as close as, or closer than, MIFIDA results for 8 out of 10 of the β and ρ parameters listed in the table. This relative efficiency of IFIVER over MIFIDA was also found in subsidiary (Bilodeau, 1978) analyses of some of the large models reported on here: IFIVER tends to converge approximately twice as fast as MIFIDA and to yield more convincing parameter values. For these reasons, and not to lengthen this long paper, we will not report further MIFIDA results.

IFIVER and MIFIDA procedures have two advantages over a FIML procedure. The first one is that very large systems can be used without some of the computing difficulties which the maximization of a likelihood function may bring for systems with large number of parameters (Models II and IV have

TABLE 1

A COMPARISON OF FIML, IFIVER AND MIFIDA RESULTS[a]

Coefficient	FIML[b]	IFIVER[c]		MIFIDA[d]	
		1[e]	2[f]	1[e]	2[f]
β_{10}	$3.68\ 10^{7}$ $(4.84\ 10^{6})$	$3.71\ 10^{7}$ $(4.92\ 10^{6})$	$3.71\ 10^{7}$ $(4.92\ 10^{6})$	$3.61\ 10^{7}$ $(4.80\ 10^{6})$	$3.61\ 10^{7}$ $(4.80\ 10^{6})$
β_{11}	$-2.00\ 10^{6}$ $(5.09\ 10^{5})$	$-2.04\ 10^{6}$ $(5.16\ 10^{5})$	$-2.04\ 10^{6}$ $(5.16\ 10^{5})$	$-2.01\ 10^{6}$ $(5.07\ 10^{5})$	$-2.01\ 10^{6}$ $(5.07\ 10^{5})$
β_{12}	$2.70\ 10^{-3}$ $(8.40\ 10^{-3})$	$2.94\ 10^{-3}$ $(8.18\ 10^{-3})$	$2.94\ 10^{-3}$ $(8.18\ 10^{-3})$	$8.21\ 10^{-3}$ $(8.13\ 10^{-3})$	$8.21\ 10^{-3}$ $(8.13\ 10^{-3})$
β_{13}	-8.80 (11.1)	-8.31 (11.0)	-8.31 (11.0)	-5.71 (11.1)	-5.71 (11.1)
β_{14}	$-4.06\ 10^{5}$ $(1.50\ 10^{5})$	$-4.14\ 10^{5}$ $(1.50\ 10^{5})$	$-4.14\ 10^{5}$ $(1.50\ 10^{5})$	$-4.09\ 10^{5}$ $(1.48\ 10^{5})$	$-4.09\ 10^{5}$ $(1.48\ 10^{5})$
β_{20}	2.64 (.256)	2.66 (.258)	2.66 (.258)	2.67 (.259)	2.67 (.259)
β_{21}	$-1.72\ 10^{-8}$ $(4.57\ 10^{-9})$	$-1.72\ 10^{-8}$ $(4.49\ 10^{-9})$	$-1.72\ 10^{-8}$ $(4.49\ 10^{-9})$	$-1.76\ 10^{-8}$ $(4.51\ 10^{-9})$	$-1.76\ 10^{-8}$ $(4.51\ 10^{-9})$
β_{22}	.685 (.081)	.680 (.082)	.680 (.082)	.681 (.082)	.681 (.082)
ρ_1	.419 (.067)	.403 (.068)	.403 (.068)	.395 (.069)	.395 (.069)
ρ_2	.880 (.030)	.881 (.030)	.881 (.030)	.881 (.030)	.881 (.030)

(a) Numbers in parentheses are standard errors.

(b) Full-information maximum likelihood.

(c) Iterated full-information instrumental variables efficient.

(d) Modified iterated full-information dynamic autoregressive.

(e) Results obtained after 25 iterations using first set of instruments for initial stage.

(f) Results obtained after 25 iterations using second set of instruments for initial stage.

respectively 104 and 106 parameters excluding those which specify error covariances across equations and standard errors). The second advantage may be that, as shown by Amemiya (1977), in a nonlinear simultaneous equations model such as (38) above, the maximum likelihood estimator requires the assumption of normality of the error term to be consistent whilst the nonlinear three stage least-squares estimator is consistent even when the normality assumption is removed: IFIVER, which solves the same set of normal equations as the 3SLSQ-like MIFIDA, should then be expected to be more robust than the FIML estimator.

We may now turn to the results obtained for our models.

4. Results

4.1. A few general remarks concerning the four models

In interpreting the results supplied in this section, one may keep in mind:

i) the parameters of all models considered were estimated from monthly time-series data for the period December 1956 to December 1971;

ii) explanatory variables were left in their "natural" dimension (dollars for the fare, minutes for waiting and in-vehicle time, etc.) whenever possible in order to be able to read directly from the coefficients the impact of unit changes in explanatory variables on the value of the dependent variable. Definitions of variables are given in the Appendix;

iii) all computations were performed with single-precision (14 significant digits) accuracy on a CDC computer: LSGAU, SSGAU and ISUGAU procedures were allowed 15 iterations which, in all cases, assured convergence of the ρ_i parameters to the fifth digit; the IFIVER procedure was allowed a sufficient number of iterations for convergence criterion (44) to be met;

iv) statistically insignificant variables were left in the equations in order not to bias the results by removing them when theoretical reasons required that they should be included.

4.2. Results for transit supply Model I

4.2.1. General comments

Model I consists of supply Equation (6), which explains the number of unit vehicle hours (of tramways, trolleybuses, buses and subway trains) supplied by the MUCTC, of congestion or cost Equation (9), which explains the average speed of those transit vehicles, and of supply/cost Equation (11), which explains the length of circuit routes and implied access costs. The equations of that S-C model were estimated by LSGAU and ISUGAU techniques, both of which assume that the explanatory variables of all equations are exogeneous to the system and the second one of which takes into account the contemporaneous correlation among error terms of the three equations. Such correlation can arise, for instance, if explanatory variables absent from all equations are correlated: it may be that bus route detours imposed by road repairs or other unspecified events jointly influence vehicle hours, speeds and route lengths. Results are shown in Tables 2, 3 and 4. A few general remarks are useful:

i) the coefficients of almost all variables have expected signs;

ii) in all cases, the "intuitive" R^2, which relates estimated errors $\hat{e}_i$ to untransformed values of the dependent variable y_i are at least as high as the R^2 which relates estimated errors to y_i^*, where the transformation is effected according to (26);

iii) the estimated value of the symmetric variance-covariance matrix of error terms $\Sigma = \Sigma_c \otimes I$ was:

$$\hat{\Sigma}_c = \begin{bmatrix} .19 \times 10^9 & -.17 \times 10^4 & -.33 \times 10^3 \\ -.17 \times 10^4 & .04 & -.08 \\ -.33 \times 10^3 & -.08 & .34 \times 10^2 \end{bmatrix},$$

which implied that the contemporaneous correlations across equations were

$$r_{12} = -.635 , \qquad r_{13} = -.004 , \qquad r_{23} = -.075 ,$$

where indices 1, 2 and 3 respectively denote the unit vehicle hours, speed and circuit length equations. As expected, errors of explanation of vehicle hours are strongly and negatively correlated with errors of explanation of speeds. Taking into account these correlations changed the signs or lowered the standard error (or both) of 25 of the 41 explanatory variables of the supply model. Although this represents a significant impact, ISUGAU results were not on the whole more convincing that LSGAU results.

4.2.2. Comments on the supply of unit vehicle hours

Results for supply Equation (6) are given in Table 2 where most variables have expected signs but not all variables are statistically as significant as might have been expected. It is interesting to note that using estimator ISUGAU rather than LSGAU changed the sign of three of the least significant variables (EX8, FIREOR and TLK). Among the factors which tend to raise revenues and lead the MUCTC to increase unit vehicle hours, demand twelve months before the current period is by far the most important one. The price elasticities[@] of supply can be computed to be 0.09 for (A) results and 0.05 for (B) results, implying a very flat supply curve because the coefficient of the variable FT, the inverse of the slope of the supply curve, is very large. The MUCTC significantly increased vehicle hours for the 1967 Fair but not for subsequent Fairs. Subsidies are very small during our sample period: it is not surprising to note that their impact is not very significant. The financial reorganization which preceded the transfer of ownership from the City of Montreal to the larger Urban Community does not have a statistically

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[@] In this paper, all elasticities are computed at the mean and have the signs of the estimated coefficients.

TABLE 2

Model I: UNIT VEHICLE HOURS SUPPLIED BY TRANSIT AUTHORITY (UVEH)

(Experiment CVEH-13-E; 15 iterations)		(A) LSGAU estimator		(B) ISUGAU estimator	
Variable	Code	Coefficient	t	Coefficient	t
EXPECTED REVENUES ($\widetilde{TR}$)					
Fare	FT	219,978.	1.39	118,341.	0.99
Seat demand lagged 12 t	XL12C	.00698	9.70	.00465	8.20
Non recurrent act. (NRA)					
. Fair in 1967	EX7	.01054	2.77	.00777	2.49
. Fair in 1968	EX8	.00341	0.25	-.01131	-1.11
. Fair in 1969	EX9	.00320	0.22	.00117	0.11
Expected subsidies ($\widetilde{SUB}$)	ESUBM	.01001	0.38	.00529	0.26
Financial reorganiz. '65	FIREOR	5,361.5	0.50	- 4,035.2	-0.50
EXPECTED COSTS ($\widetilde{UVC}$)					
Wage rate drivers	HWRD	-32,470.	-2.27	-32,550.	-2.85
Modal technology (MTCH)					
. Tramways	TR1	-18,766.	-0.88	-26,729	-1.32
. Trolleybuses	TB1	-22,647.	-1.66	-14,179.	-1.24
. Metro	STRUCT2	6,071.6	0.42	16,740	1.35
WEATHER (W)					
Warm or cold temperature	TLK	5.3066	0.35	-10.407	-0.75
Rainfall	RFK	-6,556.2	-2.57	- 9,947.6	-4.23
Snowfall	SFK	607.82	1.33	628.69	1.51
Cumulated snowfall	CSFMR	-78.697	-1.38	-36.752	-0.70
OTHER					
Transit strikes	STR	-16,045.	-30.93	-15,780.	-33.87
Metro fire '71	FIR	734.85	1.05	400.60	0.80
Weekdays/month	WD	8,559.2	6.44	9,583.3	7.97
Saturdays/month	SAT	1,931.1	0.87	3,633.3	1.81
Sundays & holidays/month	SHD	8,425.1	5.92	8,867.8	6.81
CONSTANT	C	91,250.	1.75	133,616.	2.95
SERIAL CORRELATION					
	ρ_1	.36	5.52	.46	6.82
	ρ_2	.48	7.37	.40	5.96
R-SQUARED ON TRANSFORMED y*		.88	---	.90	---
R-SQUARED ON UNTRANSFORMED y		.88	---	.90	---
DURBIN-WATSON STATISTIC		2.13	---	---	---

TABLE 3

Model I: AVERAGE SPEED OF TRANSIT VEHICLES (V)

(Experiment V-27-D; 15 iterations)		(A) LSGAU estimator		(B) ISUGAU estimator	
Variable	Code	Coefficient	t	Coefficient	t
USE OF MODES					
Seat demand by adults(SM_1)	XCANFA	-1.2714	-0.77	.54816	0.38
Seat demand by child.(SM_2)	XCSCNFA	-5.7729	-1.83	.83242	0.29
Car usage (AVE)	VNEM	-2.1160	-0.37	-6.6401	-1.41
TRANSIT VEHICLE TECHNOLOGY (VETCH)					
Share of tramway hours	SHTRH	-.31333	-0.17	.34012	0.25
Share of trolleybus hours	SHTBH	-.90971	-0.40	.57800	0.31
Share of metro hours	SHMH	11.576	14.08	11.595	16.76
WEATHER (W)					
Warm or cold temperature	TLK	-.00009	-0.39	.00013	0.57
Rainfall	RFK	.04488	1.21	.07488	2.09
Snowfall	SFK	-.01470	-2.25	-.01380	-2.12
Cumulated snowfall	CSFMR	.00060	0.62	-.00045	-0.46
OTHER					
Transit strikes	STR	-.01457	-1.07	.00503	0.41
Weekdays/month	WD	.02623	1.09	.02377	0.99
Saturdays/month	SAT	-.00180	-0.05	.01517	0.46
Sundays & holidays/month	SHD	.02383	1.02	.01778	0.73
CONSTANT	C	10.274	15.94	9.8669	15.04
SERIAL CORRELATION					
	ρ_1	.38	5.72	.39	5.50
	ρ_3	.31	4.32	.21	2.93
	ρ_{12}	.19	0.05	.27	4.69
R-SQUARED ON TRANSFORMED y*		.66	---	.70	---
R-SQUARED ON UNTRANSFORMED y		.97	---	.96	---
DURBIN-WATSON STATISTIC		2.04	---	---	---

significant impact on vehicle hours. It appears that increases of wage rates lead to significant reductions of service: the elasticity of supply with respect to drivers' wages is -0.19 in both (A) and (B). The change in the structure of costs associated with the new Metro has no clear impact on the number of unit-hours (each subway train is counted as one unit here) supplied. Among the weather variables, rainfall is the most statistically significant factor and leads to lower supply levels.

4.2.3. Comments on the determination of the average speed of vehicles

Results for C-equation (9) are given in Table 3 where most variables are not very significant. Taking into account the contemporaneous correlation in (B) changes the sign of eight of the least significant parameters without improving the results. The constant term, technological change and weather factors are by far the most significant determinants of average vehicle speeds. The peculiar autoregressive scheme of the error terms (first, third and twelfth orders) suggests the absence of explanatory variables from the equation. Passenger flows in the transit network (adults and children) and car-induced congestion do not have a very significant impact on the average speed of transit vehicles: this is not too surprising in an aggregate model.

4.2.4. Comments on the determination of route lengths

Results for C or S Equation (11) are supplied in Table 4. It is clear that the MUCTC automatically extends bus lines when urbanization increases. Sudden changes of sub-modal technology cause structural modifications of the network. Nothing of statistical significance can be drawn from a comparison of (A) and (B) results because of the quasi-independence of the errors of this

TABLE 4

Model I: CIRCUIT LENGTH OF TRANSIT VEHICLE ROUTES (CL)

(Experiment CL-4-B; 15 iterations)		(A) LSGAU estimator		(B) ISUGAU estimator	
Variable	Code	Coefficient	t	Coefficient	t
Urbanization	AR	.01515	28.13	.01534	24.16
Modal technology (MTCH)					
. Tramways	TR1	-7.1543	-0.69	-5.3706	-0.47
. Trolleybuses	TB1	6.1469	0.95	6.5954	1.01
. Metro	STRUCT2	31.404	0.84	5.2139	0.63
Trend	TREND	-7.74808	-2.49	-.84266	-2.39
Serial correlation	ρ_1	.95	99.75	.95	107.37
R-SQUARED ON TRANSFORMED y*		.47	---	.38	---
R-SQUARED ON UNTRANSFORMED y		.99	---	.99	---
DURBIN-WATSON STATISTIC		2.27	---	---	---

equation from the errors of the other equations: the very low value of these correlations explains why, in contrast with what was found for the first two equations of the model, even relatively insignificant parameters in (A) appear robust when results in (B) are examined.

4.3. Results for transit demand-supply/cost-cost Model II

4.3.1. General comments

Model II consists of adult and children transit demand Equations (14) and (15), of the modified reduced form or combined supply/cost Equation (16), which explains average transit wait-time, and of congestion or cost Equation (17), which explains average transit in-vehicle time. The equations of that D-SC-C model were estimated by LSGAU, ISUGAU and IFIVER techniques. LSGAU and ISUGAU assume that all right-

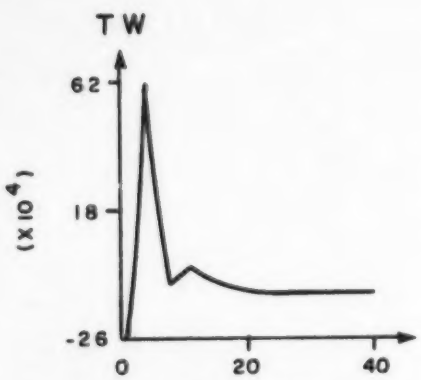
hand side variables are exogeneous but the latter takes contemporaneous correlations among the error terms of the four equations into account. In our system, summarized in (18), such correlations can arise because random or forgotten events can influence the demand and supply or cost sides simultaneously. IFIVER takes these correlations into account and further assumes that each equation contains two endogeneous (current or lagged) explanatory variables. As can be seen from Figure 1, which shows the convergence paths for the eight endogeneous variables of the system, IFIVER converged rapidly enough. Convergence was accelerated after the first two iterations by using, in the cycle, the average values of the previous pair of estimates of d in (42).

Detailed results are shown in Tables 5, 6, 7 and 8. Some general comments are useful:

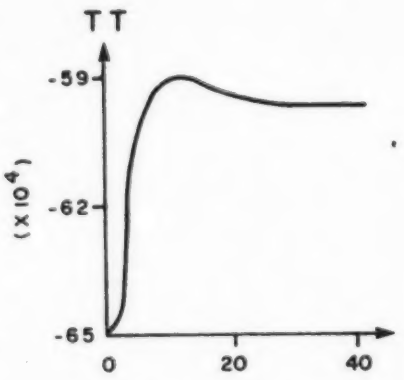
i) Examining single equation LSGAU results, the coefficients of almost all variables of the demand equations and of the aggregate congestion or cost function have expected signs. The coefficients of the waiting time equation have no straightforward interpretation because that equation is a reduced form: they are therefore of small interest for interpretative purposes which require the structural equations of Model I; we shall then refrain from commenting on them specifically in forthcoming subsections.

ii) Estimated contemporaneous correlations across equations were

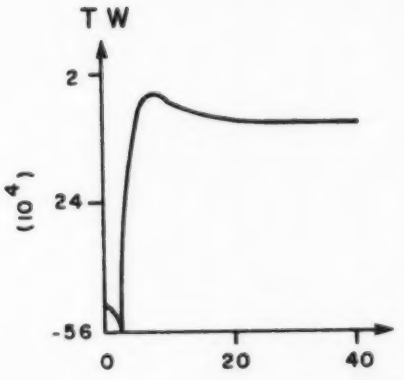
$$\begin{array}{c} \\ \text{ISUGAU} \\ \text{IFIVER} \end{array} \begin{bmatrix} r_{12} & r_{13} & r_{14} & r_{23} & r_{24} & r_{34} \\ -.23 & .24 & .39 & -.16 & -.20 & .85 \\ -.03 & -.05 & .24 & .01 & -.12 & .23 \end{bmatrix},$$



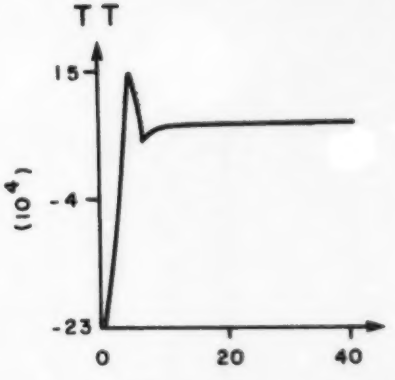
Adult demand equation
Wait-time coefficient



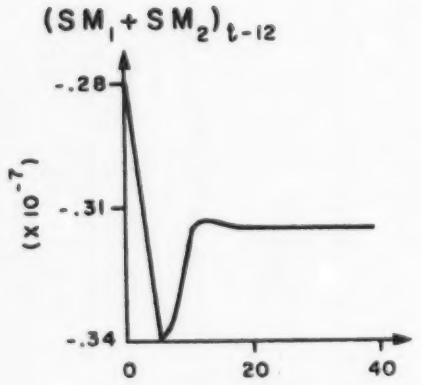
Adult demand equation
In-vehicle time coeff.



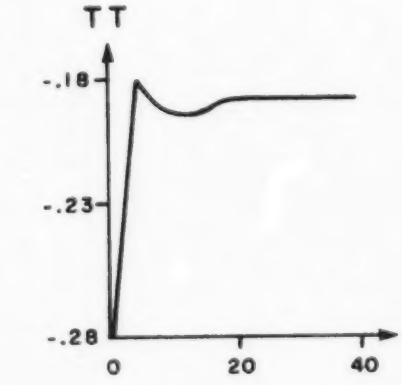
Children demand equation
Wait-time coefficient



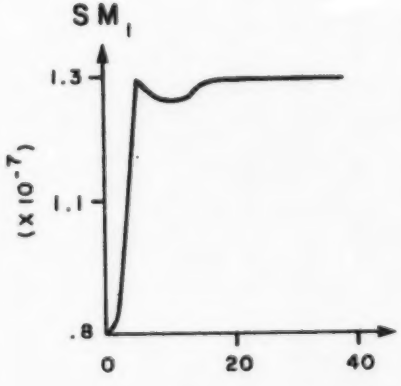
Children demand equation
In-vehicle time coeff.



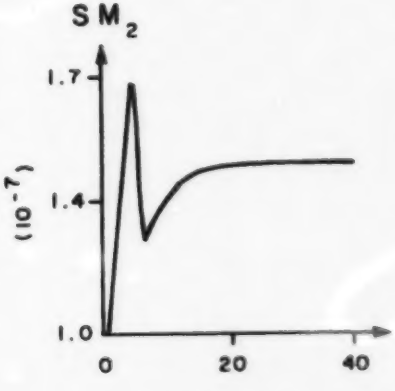
Wait-time equation
Lagged demand coefficient



Wait-time equation
In-vehicle time coefficient



In-vehicle time equation
Adult demand coefficient



In-vehicle time equation
Children demand coeff.

MODEL II : 40 ITERATIONS OF IFIVER ENDOGENEOUS VARIABLE COEFFICIENT ESTIMATES

FIGURE 1

TABLE 5

MODEL II: DEMAND FOR TRANSIT BY ADULTS (SM_1)

(Experiment XA 2-17-61-C)		(A) LSGAU estimator*		(B) ISUGAU estimator*		(C) IFIVER estimator**	
Variable	Code	Coefficient	t-stat.	Coefficient	t-stat.	Coefficient	t-stat.
P:Real transit fare	FAD	-22,402,011.	-6.44	-24,096,391.	-7.05	-27,374,140.	-6.95
Number of cars	NC	-22.706	-3.11	-20.838	-2.91	-23.957	-3.01
Consum.price index	CPI	14,052.	0.37	1,460.	0.04	29,123.	0.57
Y:Real average earn.	RAWEM	134,819.	3.69	127,031.	3.55	143,597.	3.56
C:Transit wait-time	TW	-823,735.	-2.12	-715,267.	-1.87	-123,851. ***	-0.20
Trans.in-veh.time	TT	-467,189.	-12.74	-508,690.	-14.00	-600,219. ***	-11.97
Car in-veh.time	T	146,578.	0.42	177,383.	0.50	-327,333.	-0.67
W:Temperature discomf.	TLK	346.65	0.43	465.10	0.59	880.82	1.08
Rainfall discomf.	RFK	-220,298.	-2.98	-225,593.	-3.07	-208,828.	-2.38
Snowfall discomf.	SFK	1,681.	0.11	2,055.	0.14	10,222.	0.58
Cumul.snowf.discomf.	CSFMR	3,290.	1.23	3,334.	1.27	3,799.	1.31
A:Job presence index	EANFP	.03622	5.39	.03315	5.21	.03241	5.49
College-univ.index	RCUTP	-.57248	-0.07	1.2303	0.17	10.073	1.50
Shopping index	SHONF	5.5540	3.13	5.0308	2.96	4.7650	2.96
Fair '67	EX7	2.6476	28.20	2.6410	28.60	2.7183	24.73
Fair '68	EX8	2.4615	6.44	2.2743	6.07	2.2524	5.67
Fair '69	EX9	1.7531	2.95	1.5809	2.73	1.5506	2.60
Fair '70	EX0	.60568	0.77	.33153	0.43	.13041	0.17
Fair '71	EX1	.31577	0.34	-.01090	-0.01	-.05800	-0.06
Transit strikes	STR	-726,854.	-51.27	-732,589.	-51.93	-748,854.	-46.04
Fire metro '71	FIR	-48,304.	-2.11	-47,440.	-2.13	-39,087.	-1.56
ETC:Anticipatory sales	ANT	-993,573.	-2.72	-911,993.	-2.55	-837,986.	-2.18
Ant.sales lagged one	ANTL1	58,118.	0.15	381,841.	1.03	590,996.	1.52
Zonal double count.	ZA	6,292,448.	2.95	5,737,907.	2.75	6,508,274.	2.63
Weekdays/month	WD	657,245.	6.01	648,140.	6.17	575,921.	5.62
Saturdays/month	SAT	456,308.	3.69	435,009.	3.63	319,926.	2.48
Sun.+ holidays/m.	SHD	353,979.	3.06	349,964.	3.14	288,554.	2.62
Constant	C	7,093,029.	1.21	10,102,138.	1.78	12,562,290.	2.02
Serial correlation	ρ_1	.12	2.71	.13	2.81	.13	2.38
	ρ_{12}	.74	17.05	.73	16.36	.63	12.13
R-SQUARED ON TRANSFORMED y*		.97	---	.97	---	n.c.	---
R-SQUARED ON UNTRANSFORMED y		.97	---	.97	---	n.c.	---
DURBIN-WATSON STATISTIC		1.65	---	1.65	---	n.c.	---

* 15 iterations. ** 40 iterations. *** Treated as endogeneous variable.

TABLE 6

MODEL II: DEMAND FOR TRANSIT BY CHILDREN (SM_2)

(Experiment XSC 2-17-118-N)		(A) LSGAU estimator*		(B) ISUGAU estimator*		(C) IFIVER estimator**	
Variable	Code	Coefficient	t-stat.	Coefficient	t-stat.	Coefficient	t-stat.
P:Real transit fare	FSCD	-22,597,794.	-3.75	-22,413,647.	-3.70	-27,142.170.	-3.89
Number of cars	NC	-11.484	-2.30	-10.972	-2.16	-13.171	-2.45
Consum.price index	CPI	11,832.	0.36	8,648.1	0.26	48,677.	1.34
Y:Real average earn.	RAWEM	25,341.	1.39	25,700.	1.38	21,018.	1.09
C:Transit wait-time	TW	-166,420.	-0.79	-228,067.	-1.00	-61,635.***	-0.22
Transit in-veh.time	TT	-22,620.	-0.52	20,071.	0.46	99,115.***	1.26
Car in veh.time	T	136,326.	0.48	159,823.	0.55	-141,871.	-0.48
W:Temperature discomf.	TLK	1,067.	3.15	833.	2.55	500.	1.31
Rainfall discomf.	RFK	-51,312.	-1.19	-49,840.	-1.16	-72,509.	-1.60
Snowfall discomf.	SFK	-17,069.	-2.10	-18,877.	-2.32	-13,306.	-2.01
Cumul.snowf.discomf.	CSFMR	1,319.	1.02	1,560.	1.18	2,414.	1.71
A:School index	RSCMP	3.3129	8.06	3.2673	7.84	3.1171	7.47
Shopping index	SHONF	1.5854	2.86	1.5050	2.63	1.4179	2.47
Fair '67	EX7	.19312	4.75	.18823	4.50	.15361	3.29
Fair '68	EX8	.24043	1.55	.22175	1.40	.16957	1.05
Transit strikes	STR	-118,462.	-13.15	-118,173.	-13.13	-111,679.	-11.84
Trend	TEND	19,538.	2.72	19,666.	2.69	14,488.	1.96
ETC:Anticipatory sales	ANT	1,254,945.	5.78	1,220,156.	5.67	1,161,559.	5.56
Ant.sales lagged one	ANTL1	-430,819.	-1.78	-440,112.	-1.82	-400,948.	-1.70
Zonal double count.	ZSC	2,486,680.	3.70	2,792,077.	4.06	3,642,174.	4.51
Weekdays/month	WD	100,126.	2.92	109,166.	3.12	108,969.	3.06
Saturdays/month	SAT	37,424.	0.80	47,977.	1.02	38,056.	0.77
Sun.+ holidays/m.	SHD	32,644.	0.88	43,797.	1.16	34,032.	0.88
Fare card control	CHEAT2	-64,535.	-0.43	-72,554.	-0.48	-21,935.	-0.14
Line restructuration	STRUCT1	-257,829.	-0.98	-53,739.	-0.20	429,841.	1.00
Student card exp.	CARDEX	-107,921.	-0.68	-116,989.	-0.72	-127,225.	-0.80
Constant	C	-1,651,519.	-0.64	-2,634,091.	-1.01	-5,805,038.	-1.99
Serial correlation	ρ_1	.16	2.50	.18	2.80	.21	3.28
	ρ_3	-.31	-4.86	-.27	-4.19	-.27	-4.20
	ρ_4	-.06	-0.98	-.07	-1.04	-.09	-1.34
	ρ_{12}	.41	6.37	.42	6.85	.40	6.30
R-SQUARED ON TRANSFORMED y*		.84	---	.83	---	n.c.	---
R-SQUARED ON UNTRANSFORMED y		.93	---	.93	---	n.c.	---
DURBIN-WATSON STATISTIC		1.91	---	1.92	---	n.c.	---

* 15 iterations. ** 40 iterations. *** Treated as endogeneous variable.

TABLE 7

MODEL II: TRANSIT WAITING TIME (TW)

(Experiment TW-2-L)		(A) LSGAU estimator*		(B) ISUGAU estimator*		(C) IFIVER estimator**	
Variable	Code	Coefficient	t-stat.	Coefficient	t-stat.	Coefficient	t-stat.
VARIABLES FROM STRUCTURAL EQ. (6)							
$\tilde{TR}$:Fare	FT	-.44419	-0.55	-.26636	-0.34	-.30660	-0.44
Seat dem. lagged 12 t	XL12C	-.40136	-11.56	-.39429	-11.38	-.31438****	-9.63
Fair '67	EX7	-1.0790	-6.08	-1.0721	-6.12	-.65071	-3.33
Fair '68	EX8	.26796	0.44	-.08162	-0.14	-1.6507	-3.43
Fair '69	EX9	.41592	0.61	.21094	0.31	-.81527	-1.43
Expected subsidies	ESUBM	-1.42637	-1.06	-1.1079	-0.84	.05006	-0.04
Financial reorg.'65	FIREOR	.14820	2.49	.14148	2.41	.10871	2.23
$\tilde{TC}$:Wage rate drivers	HWRD	.40135	5.05	.40257	5.10	.25614	3.29
Tramway technol.	TR1	.01294	0.11	.11360	0.93	.19921	0.68
Trolleybus technol.	TB1	.12678	1.84	.13021	1.89	.14517	2.05
Metro technol.	STRUCT2	-.45848	-4.87	-.53329	-5.67	-.88203	-7.03
W:Warm or cold temp.	TLK	-.00004	-0.55	-.00005	-0.71	-.00004	-0.41
Rainfall	RFK	.03178	2.56	.03433	2.64	.02039	1.18
Snowfall	SFK	.00097	0.41	.00177	0.74	.00269	0.81
Cumul. snowfall	CSFMR	.00044	1.48	.00049	1.64	.00058	1.39
ETC:Transit strikes	STR	.00729	2.64	.00753	2.73	.00582	1.59
Metro fire '71	FIR	-.00612	-1.74	-.00630	-1.81	-.00672	-2.39
Weekdays/month	WD	.03202	5.08	.02841	4.36	.01609	1.82
Saturdays/month	SAT	.08989	8.17	.08668	7.56	.07270	4.79
Sun.+ holidays/m.	SHD	.04603	5.74	.04547	5.55	.03754	3.59
VARIABLES FROM STRUCTURAL EQ. (8)							
Length of routes or circuits	CL	.00304	5.06	.00275	4.18	.00128	2.18
Transit travel time	TT	-.06327	-4.47	-.08684	-6.18	-.19141***	-7.97
OTHER VARIABLES							
Collective agreement	CLAG	.00835	0.10	.01751	0.22	.05803	0.89
Constant	C	1.4200	2.54	2.2763	3.75	6.9194	8.13
SERIAL CORRELATION	ρ_1	.44	7.60	.43	7.73	.50	8.86
	ρ_3	.44	7.88	.47	8.99	.45	8.28
R-SQUARED ON TRANSFORMED y*		.77	---	.76	---	n.c.	---
R-SQUARED ON UNTRANSFORMED y		.98	---	.98	---	n.c.	---
DURBIN-WATSON STATISTIC		1.96	---	1.95	---	n.c.	---

* 15 iterations. ** 40 iterations. *** Treated as current endogeneous variable.

**** Treated as lagged endogeneous variable.

TABLE 8

MODEL II: TRANSIT IN-VEHICLE TRAVEL TIME (TT)

(Experiment TT-15-F)		(A) LSGAU estimator*		(B) ISUGAU estimator*		(C) IFIVER estimator **	
Variable	Code	Coefficient	t-stat.	Coefficient	t-stat.	Coefficient	t-stat.
USE OF MODES							
Seat dem.adults(SM_1)	XCANFA	.48009	1.29	.35248	0.98	1.2570***	3.74
Seat dem.child.(SM_2)	XCSCNFA	.09745	1.38	1.8225	2.74	1.4493***	2.99
Car usage (AVE)	VNEM	.02453	0.19	.03912	0.32	-.07740	-1.07
TRANSIT VEHICLE TECHNOLOGY (VETCH)							
Share of tramways hrs	SHTRH	1.0356	0.26	2.0702	0.52	9.3597	1.69
Share of trolleyb.hrs	SHTBH	3.8275	0.71	5.5147	1.08	5.9495	1.67
Share of metro hours	SHMH	-22.336	-11.18	-21.876	-11.22	-23.110	-12.35
WEATHER (W)							
Warm or cold temp.	TLK	.00023	0.46	.00039	0.78	.00063	1.32
Rainfall	RFK	-.08169	-0.92	-.05067	-0.57	-.00515	-0.06
Snowfall	SFK	.02577	1.63	.02675	1.72	.01683	1.14
Cumul.snowfall	CSFMR	-.00123	-0.54	-.00192	-0.88	-.00240	-1.22
OTHER							
Transit strikes	STR	.04485	1.44	.04013	1.33	.09580	3.32
Weekdays/month	WD	-.09294	-1.64	-.07528	-1.36	-.10751	-2.19
Saturdays/month	SAT	-.00192	-0.02	.02218	0.27	-.00150	-0.02
Sun.+ holidays/month	SHD	-.05881	-1.04	-.05727	-1.02	-.6433	-1.28
CONSTANT	C	24.107	15.57	23.455	15.49	23.219	16.26
SERIAL CORRELATION	ρ_1	.32	4.34	.31	4.26	.34	4.66
	ρ_2	.25	3.30	.24	3.26	.25	3.26
	ρ_3	.19	2.52	.20	2.76	.20	2.71
	ρ_{12}	.15	2.85	.14	2.70	.10	2.11
R-SQUARED ON TRANSFORMED y*		.54	---	.53	---	n.c.	---
R-SQUARED ON UNTRANSFORMED y		.94	---	.94	---	n.c.	---
DURBIN-WATSON STATISTIC		1.99	---	1.98	---	n.c.	---

* 15 iterations. ** 40 iterations. *** Treated as endogeneous variable.

where indices respectively denote adult demand, children demand, wait-time and travel-time equations. When ISUGAU results are compared to LSGAU results, 50 of the 94 explanatory variables of the system have lower standard errors and 5 variables of low statistical significance (RCUTP and EX1 in Table 5, TT in Table 6, EX8 in Table 7 and SAT in Table 8) change signs. Explicit recognition of the simultaneous nature of the system with the IFIVER procedure changes the signs of 5 more variables of low statistical significance (T in Table 5, T and STRUCT1 in Table 6, EX9 in Table 7 and VNEM in Table 8). ISUGAU results for either the demand side of the supply-cost side considered separately are available to the reader upon request.

4.3.2. Comments on the transit demand equations

As explained in Section 1.3, it is of great interest to try to remove simultaneous equations biases from estimates of travel demand equations. Removing such biases from service level variable coefficients might change the ranking of price and service level elasticities and influence the implicit money values of time derived from the results. The main results derived from Table 5 and Table 6 are found in Table 9.

We note the following:

i) As one compares (A) and (B) columns, grouping equations for estimation purposes reverses the sign of the travel time variable in the children equation but that variable is not very significant;

ii) As expected, removing the simultaneous equations biases in (C), as compared to (B), increases fare elasticities relatively to service level elasticities and lowers the wait-time elasticities relative to

TABLE 9

MODEL II: ADULT AND CHILDREN ELASTICITIES* OF DEMAND AND TIME PRICES

	A. LSGAU		B. ISUGAU		C. IFIVER	
	el.	$/hr	el.	$/hr	el.	$/hr
Adults : fare	-.18 (6.44)	--	-.19 (-7.05)	--	-0.22 (-6.95)	--
wait-time	-.16 (-2.12)	$2.20	-.14 (-1.87)	$1.78	-0.02 (-0.20)	$0.27
in-vehicle time	-.49 (-12.74)	$1.25	-.53 (-14.00)	$1.27	-.63 (-11.97)	$1.32
Children: fare	-.44 (-8.06)	--	-.43 (-3.70)	--	-.52 (-3.89)	--
wait-time	-.21 (-0.79)	$0.44	-.29 (-1.00)	$0.61	-.07 (-0.22)	$0.14
in-vehicle time	-.16 (-0.52)	$0.06	.14 (0.46)	-$0.05	.69 (1.26)	-$0.22

* Numbers in parentheses are t-statistics

the travel time elasticities: the latter result is completely in keeping with the expectation that wait-time elasticities are biased upwards and travel-time elasticities are biased downwards if limited-informations procedures (A) and (B) are used. Unfortunately wait-time becomes insignificant: the cure in (C) is as bad as the disease;

iii) the implicit values of time obtained by a full-information technique in (C) are very different from those obtained by a limited-information technique in (A) but one cannot be satisfied with the (C) results: the model may include problems of specification which surface in (C).

4.3.3. Comments on the transit congestion or cost equation

The most interesting results in Table 8 are that full-information results in (C) increase the statistical significance of endogeneous variables.

More precisely, these results imply that usage of transit significantly slows vehicles down. The "congestion elasticities" are 0.12 for adults and 0.02 for children. Given that children represent only 15% of riders, this means that children cause relatively more congestion than adults, no doubt because of a less uniform travel frequency distribution during the day.

4.4. Results for car demand-cost model III

4.4.1. General comments

Model III consists of a gasoline or car usage demand equation and of an aggregate automobile congestion or cost function. Although the structure of that two-equation system (19) is a simultaneous structure in which car usage depends on travel time by car, and conversely, we will not use full-information estimation techniques because the poor quality of our observations on average monthly car travel times means that our estimate of the error term of the congestion equation is certainly biased. In those circumstances, IFIVER (and ISUGAU) would reverberate on the system a very biased estimate of contemporaneous correlations among equations. It is preferable to neglect the information contained in the contemporaneous covariance matrix and to use single equation techniques. In consequence, the gasoline demand equation was estimated with LSGAU and SSGAU techniques, the latter of which takes into account the endogeneous nature of the congestion variable (T). The specification of the congestion equation was not judged satisfactory enough for the use of a more sophisticated technique than LSGAU to be appropriate. A number of remarks can be made about the results.

4.4.2. Comments on the gasoline or car usage demand equation

The results obtained for the gasoline demand equation are shown in Table 10. In column (A), all variables have expected signs except the price of gasoline (AUMID), the Fairs in 1970 and 1971 (EX0 and EX1) and, perhaps, total school activity (TRCUSP); among those, however, only the last one is statistically significant. We have no particular expectation concerning the effect of congestion (T), which simultaneously increases gasoline consumption per trip and reduces trip frequency, although we suspect that the former effect should be larger than the latter (implying a positive sign for travel time variable T). As can be seen by comparing columns (A) and (D), using the two-stage-least-squares technique SSGAU improved the results considerably because

i) the statistical significance of three of the variables which have unexpected signs (EX0, EX1 and TRCUSP) decreased and the sign of the fourth one (AUMID) changed in the anticipated direction;

ii) the sign of the travel time variable T also changed in the suspected anticipated direction (the correlation between values of T used in (A) and (D) was 0.97);

iii) the statistical significance of 17 of the remaining 21 parameters improved.

Demand elasticities of particular interest are shown in Table 11. If one accepts that increases in automobile congestion increase total consumption of gasoline, all of the elasticities are reasonable. Given the knowledge that gasoline is a superior good (the sign of income variable RAWEM is positive), the high elasticity of demand with respect to the consumer

TABLE 10

MODEL III: DEMAND FOR GASOLINE OR CAR USAGE (AVE)

(Experiment VNEM-4-C; 15 iterations)		(A) LSGAU estimator		(D) SSGAU estimator	
Variable	Code	Coefficient	t-statistic	Coefficient	t-statistic
PRICES (P)					
Real cost of gasoline	AUMID	1,707.	0.01	-19,054.	-0.17
Number of cars	NC	44.336	1.26	43.986	1.25
Real transit adult fare	FAD	21,168,172.	1.05	13,470,730.	0.61
Consumer price index	CPI	13,776.	0.06	-102,086.	-0.47
INCOME (Y)					
Real average earnings	RAWEM	149,382.	0.65	2,246.	0.01
COSTS OR SERVICE LEVELS (C)					
Car in-vehicle time	T	-1,166,933.	-0.43	1,174.850.*	0.60
Transit wait-time	TW	1,726,782.	0.71	2,208,867.	0.93
Transit in-vehicle time	TT	169,717.	0.78	196,519.	0.89
WEATHER (W)					
Temperature discomfort	TLK	-4,617.	-1.38	-5.108.	-1.58
Rainfall discomfort	RFK	-670,822.	-1.80	-658,314.	-1.72
Snowfall discomfort	SFK	135,979.	1.89	144,447.	1.98
Cumulated snowfall discomfort	CSFMR	-30,333.	-2.48	-32,556.	-2.70
ACTIVITIES (A)					
Employment level	EI	151,398.	1.84	239,425.	2.60
Workers' vacations	VWM	212,994.	1.45	257,432.	1.87
Total school index	TRCUSP	-11.434	-3.22	-10.313	-2.87
Shopping index	SHONF	31.448	5.11	30.821	5.81
Fair '67	EX7	1.2934	2.51	1.4351	2.64
Fair '68	EX8	3.7107	1.89	4.0940	2.07
Fair '69	EX9	3.8511	1.40	3.9997	1.48
Fair '70	EX0	-2.2179	-0.65	-2.0001	-0.61
Fair '71	EX1	-0.4928	-0.13	-0.2937	-0.01
Transit strikes	STR	153,329.	2.03	161,461.	2.09
OTHER					
Weekdays/month	WD	490,386.	1.67	530,004.	1.91
Saturdays/month	SAT	40,877.	0.09	38,290.	0.10
Sundays & Holidays/month	SHD	96,564.	0.29	100,244.	0.32
CONSTANT	C	-29,162.150.	-1.12	-36,655.700.	-1.51
SERIAL CORRELATION	ρ_1	.31	5.07	.31	4.97
	ρ_{12}	.46	7.39	.40	6.71
R-SQUARED ON TRANSFORMED y*		.68	---	.75	---
R-SQUARED ON UNTRANSFORMED y		.98	---	.97	---
DURBIN-WATSON STATISTIC		2.02	---	2.02	---

* Treated as endogeneous variable.

TABLE 11

MODEL III: SELECTED GASOLINE OWN AND CROSS ELASTICITIES* OF DEMAND

	Own elasticities	Cross elasticities	
Real price of gasoline	-0.05	Real adult transit fare	0.07
Car in-vehicle travel time	0.35	Transit in-veh. travel time	0.12
---	--	Transit waiting time	0.26
Number of cars	0.40	---	--
---	--	Consumer price index	0.42

*Computed from SSGAU estimates.

price index implies that gasoline is a strong substitute for other goods than transportation. Its own price elasticity of demand, however, is low. The results show the high impact of transit service levels on the use of the automobile.

Other results of interest in Table 10 are the significant impact of weather variables and of transit strikes on gasoline consumption.

4.4.3. Comments on the automobile congestion or cost equation

The results obtained for the aggregate automobile congestion equation are shown in Table 12. We did not take serial correlation into account because of the poor quality of our data on the dependent variable. The low value of the Durbin-Watson statistic implies that our standard errors, and the resulting t-statistics, are probably biased upwards. Although the results cannot be interpreted with the same degree of confidence as those supplied for all other equations of this paper, they indicate the importance of street management and imply that higher gasoline prices induce drivers to drive more slowly on average, thereby increasing observed travel times separately from other factors such as greater usage of streets.

TABLE 12

MODEL III: AUTOMOBILE IN-VEHICLE TRAVEL TIME (T)

(Experiment T2T-3)		(A) LSGAU estimator	
Variable	Code	Coefficient	t-statistic
USE OF MODES			
Car usage (AVE)	VNEM	.03796	5.68
DRIVERS' BEHAVIOR			
Real price of gasoline	AUMID	.04261	4.68
NETWORK MANAGEMENT			
One-way streets	TOWN	-.00924	-7.50
WEATHER (W)			
Warm or cold	TLK	.00019	0.41
Rainfall	RFK	-.07038	-0.74
Snowfall	SFK	.00674	0.39
Cumulative snowfall	CSFMR	.00292	1.45
CONSTANT	C	5.44	5.48
R-SQUARED ON UNTRANSFORMED y		.55	--
DURBIN-WATSON STATISTIC		.16	--

4.5. Results for bi-modal demand-supply/cost-cost Model IV

Model IV, summarized in (21), consists of a grouping of equations taken from the previous two models. It includes

i) the gasoline demand equation of Table 10;

ii) a total transit demand equation constructed by integrating regressor lists of Tables 5 and 6;

iii) the reduced form equation for transit waiting time shown in Table 7;

iv) the aggregate transit congestion equation shown in Table 8 and modified slightly by grouping adult and children use of transit variables.

The automobile congestion equation has not been included in the system because of its previously mentioned insufficiencies and in order not to enlarge the system unduly: Model IV contains 97 explanatory variables and 9 autoregressive parameters; each equation has two endogeneous explanatory variables.

We will not reproduce the full results for Model IV because they would not add much to results reported for Models II and III. The full results are available to the reader upon request. Suffice is to say that, when single-equation estimation technique LSGAU was used, results of the aggregated transit demand equation were not significantly different from those already shown for adults and children considered separately. Using aggregate transit usage instead of distinct usage by adults and children had no significant impact on the explanation of transit in-vehicle time already presented in Table 8.

However, as the purpose of Model IV is to mesure the effect of a simultaneous explanation and estimation of travel demands by mode and of transit service levels, we present in Table 13 selected results for the endogeneous explanatory variables. Remembering that Equation [S/C] is a reduced form equation rather than a structural equation (so that the signs of its parameters have no straightforward interpretation), we note the following points of interest:

i) As in Model II, a comparison of columns (A) and (B) shows that taking contemporaneous correlations of errors among equations into account reduces standard errors of estimates; similarly, a comparison of columns (B) and (C) shows that removing the simultaneous equations

TABLE 13

MODEL IV: SELECTED RESULTS FOR THE ENDOGENEOUS VARIABLES OF THE FOUR EQUATIONS

Equation identification Endogeneous explanatory variable Variable code	(A)LSGAU estimator*		(B) ISUGAU estimator*		(C) IFIVER estimator**	
	Coefficient	t-stat.	Coefficient	t-stat.	Coefficient	t-stat.
D-1. DEMAND FOR GASOLINE OR CAR USAGE (AVE)						
(Experiment VNEM-4-C)						
...						
Transit wait-time TW	1,726,782.	0.71	884,657.	0.36	-1,678,107.***	-0.48
Transit in-vehicle time TT	169,717.	0.78	275,014.	1.27	548,885.***	1.80
...						
D-2. TOTAL DEMAND FOR TRANSIT ($SM_1 + SM_2$)						
(Experiment X-3-R)						
...						
Transit wait-time TW	-283,057.	-0.81	-337.010.	-0.96	-209,223.***	-0.25
Transit in-vehicle time TT	-455,513.	-11.86	-472,960.	-12.31	-604,677.***	-10.00
...						
S/C. WAITING TIME FOR TRANSIT (TW)						
(Experiment TW-2-L)						
...						
Seat demand lagged 12 t XL12C	-.40136	-11.56	-.39504	-11.42	*** -.1907	-7.84
Transit in-vehicle time TT	-.06327	-4.47	-.08849	-5.93	*** -.07204	-9.80
...						
C. TRAVEL TIME BY TRANSIT (TT)						
(Experiment TT-16-F)						
...						
Total seat demand (SM_1+SM_2) X	.69336	2.45	.88042	3.17	*** 1.4915	5.57
Car usage (AVE) VNEM	-.02156	-0.02	.14448	0.14	*** -.42045	-0.51
...						

* 15 iterations. ** 40 iterations. *** Treated as endogeneous variable.

bias from demand Equations [D-1] and [D-2] increases the relative importance of transit in-vehicle time compared to transit wait time. The implied demand elasticities are shown in Table 14. ISUGAU results for either the demand side or the supply-cost side considered separately are available to the reader upon request.

TABLE 14

MODEL IV: ELASTICITIES OF DEMAND

Market	Transit variable	(A) LSGAU	(B) ISUGAU	(C) IFIVER
For gasoline	Wait-time	.20	.10	-.19
	In-vehicle time	.11	.18	.35
For transit	Wait-time	-.05	-.06	-.04
	In-vehicle time	-.41	-.43	-.56

ii) As in Model II, the fact that transit travel time is statistically more significant than transit wait-time may be due to the fact that fluctuations of the in-vehicle time series are in great part due to the substitution of metro line for bus lines: this not only increases transit speed but also improves transit comfort, a factor which was not accounted for in the analysis.

iii) Removing the simultaneous equations bias from Equation [C] roughly doubles the measure of congestion created by passenger flows ($SM_1 + SM_2$) in the network.

CONCLUSION

This paper has presented formulations and estimates of the parameters of transit supply, demand and congestion functions and of automobile demand and congestion functions. We have emphasized the differences between supply functions, which describe how economic agents determine the services they offer, and congestion or cost functions, which describe how some travel time costs are determined by the use of travel modes, technology and other factors.

We have shown that the results obtained with single equation estimation techniques (LSGAU and SSGAU) can differ significantly from those obtained by grouping equations for estimation purposes (ISUGAU) or from those obtained from full-information procedures (IFIVER, MIFIDA and FIML). Notably, we have pointed out that, at least in aggregate travel demand equations, simultaneous equations biases tend to increase waiting time elasticities relative to travel time elasticities and to affect the relative importance of price and service level explanatory variables and of derived values of time. The results tend to confirm the initial supposition that simultaneous equations biases can produce incorrect estimates of important parameters, perhaps causing ill-advised policy recommendations; but the results also suggest that using full-information estimation methods does not automatically yield unambiguously preferable results because functional form or other sources of mispecification of any of the relationships of the system colour system results.

We have examined the behavior of a regulated transit authority to bring

out the importance of solving the supplier's own equilibrium problem simultaneously with the demand-cost equilibrium problem. We have hopefully shown that D-S systems are conceptually and practically different from D-C systems often used in urban contexts. A D-S-C model structure has been used to describe the multiple simultaneous equilibrium processes involved. We have not commented at length on the importance of the transit operator's *lagged* adjustment of supply but it should be clear that D-S-C structures need not describe only *contemporaneous* equilibria to be useful tools of analysis.

ACKNOWLEDGEMENTS

The Research and Development Centre of Transport Canada and the F.C.A.C. program of the *Ministère de l'éducation du Québec* have supported the research on which this article is based. Marcel G. Davenais of *L'Université de Montréal* , provided valuable advice and Ernest R. Berndt, of the University of British Columbia, made useful comments on the estimation techniques. Ray C. Fair, of Yale University, supplied the FIML program and gave precisions on some features of his FIVER estimator. Yvon Sabourin, Danielle Bilodeau and Tram Cong Liem performed computations. The Montreal Urban Community Transportation Commission and various government agencies supplied unpublished data. The author is solely responsible for any misuse of these resources. Preliminary results were presented at the International Symposium on Travel Supply Models, *Université de Montréal* , November 1977. Final results were presented at the Workshop in Analytical Urban Economics, Queen's University, June 1978. Donald N. Dewees, of the University of Toronto, made detailed suggestions which had a significant influence on the final text.

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APPENDIX: DEFINITIONS OF VARIABLES

Note that variables are listed alphabetically by their code names. Expository names are in parentheses. The symbol "c.f." means "constructed from". Many "dummy" variables include changes of level incorporating *a priori* information. All variables are defined per month.

DEPENDENT VARIABLES

CL : circuit length in miles or sum of route lengths of tramways, trolleybuses and buses.

T : travel time by car in minutes c.f. representative 4-mile distance divided by car speed.

TT : transit travel time in minutes or representative 4-mile distance divided by average transit vehicle commercial speed V.

TW : transit wait-time in minutes or half of headway c.f. UVEH, CL and total number of strikeless days per month.

UVEH : unit vehicle hours or sum of service hours of tramways, trolleybuses, buses and subway trains.

V : average commercial speed c.f. vehicle-miles and vehicle-hours measured by hours worked by drivers.

VNEM : (AVE) gasoline consumption in gallons c.f. provincial monthly sales multiplied by Montreal's quarterly share (using 1970 shares before 1967). Excludes diesel fuel sales.

X : (SM) number of transit trips or sum of collected tickets and cash transformed into trips.

XCANFA : (SM_1) number of adult trips obtained by multiplication of X by share of adult tickets in total ticket sales.

XCSCNFA : (SM_2) number of children trips obtained by multiplication of X by share of children tickets in total ticket sales.

INDEPENDENT VARIABLES

ANT : dummy variable to account for unusual behavior of total ticket sales before a fare increase in 1965.

ANTL1 : same as ANT, but lagged one period.

AR : number of acres of urbanized territory served by the MUCTC.

AUMID : price of regular gasoline in Montreal deflated by CPI.

C : constant.

CARDEX : dummy variable to account for the summer expiration of certain students' reduced price cards since 1963.

CHEAT2 : dummy variable to account for the introduction in November 1970 of computer control of information supplied in reduced fare card applications.

CLAG : dummy variable to account for new way of counting hours worked when vehicles enter or leave a garage since collective agreement of 1965.

CPI : consumer price index for Montreal.

CSFMR : cumulated snowfall every winter until March; one quarter of the March value for April and zero until following winter.

EANFP : the sum of work origins and destinations in the territory served by the MUCTC weighted by EI multiplied by a non-vacation index.

EI : employment index for Montreal.

ESUBM : expected monthly subsidy at budget time.

EX_ : the number of passengers who enter the metro station situated on the site of the fairs held in 1967, 1968, 1969, 1970, 1971.

FAD : the unit price of quantity discounted adult tickets deflated by CPI.

FIR : the number of days during which the lines were temporarily reorganized following metro fires.

FIREOR : dummy variable to represent the financial reorganization in 1965 and 1966 before the transfer of ownership from the City of Montreal to the regional government.

FSCD : the unit price of quantity discounted children tickets deflated by CPI.

FT : an index of the average non deflated ticket fares for adults and children c.f. annual market shares.

HWRD : drivers' wage rate.

NC : total number of cars owned in area served by MUCTC.

RAWEM : average weekly earnings in Montreal deflated by CPI.

RCUTP : number of college and university students above 18 years of age multiplied by a non-vacation index; c.f. registrations and calendars of individual institutions or groups of institutions.

RFK : average daily rainfall.

RSCMP : number of school children less than 18 years of age multiplied by a non-vacation index, c.f. registrations and calendars of primary schools, highschools and colleges.

SAT : number of Saturdays.

SFK : average daily snowfall.

SHD : number of Sundays and Holidays.

SHMH : share of metro vehicle hours in total vehicle hours.

SHONF : sum of provincial department store and family clothing sales deflated by CPI and modified to correspond to territory served by the MUCTC.

SHTBH : share of trolleybus vehicle hours in total vehicle hours.

SHTRH : share of tramway vehicle hours in total vehicle hours.

STR : number of days of total shutdown of service during transit strikes.

STRUCT1 : dummy variable to represent the first two metro lines.

STRUCT2 : dummy variable to represent all metro lines.

TB1 : dummy variable to represent the existence of trolleybus lines.

TEND : linear trend reaching a maximum in October 1969 and falling afterwards to represent the population of children.

TLK : temperature discomfort index defined as the absolute value of difference between the mean temperature of the current month and of a representative September.

TOWN : dummy variable to represent increasing use of one-way streets over a part of the sample period.

TR1 : dummy variable to represent existence of tramway lines.

TRCUSP : school activity index consisting of sum of RCUTP and RSCMP.

TREND : linear trend.

VWM : number of vacation days of average worker.

WD : number of work days.

XL12C : (SM_{t-12}) total number of passengers lagged twelve periods and corrected to remove impact of 1967 and 1968 fairs.

ZA : ratio of adult ticket sales in outside fare zones to total adult ticket sales.

ZSC : ratio school ticket sales in outside fare zones to total school ticket sales.